1.1 Identify Points, Lines, and Planes

Goal • Name and sketch geometric figures.

Your Notes

VOCABULARY

Undefined term A word without a formal definition

Point A point has no dimension. It is represented by a dot.

Line A line has one dimension. It is represented by a line with two arrowheads.

Plane A plane has two dimensions. It is represented by a shape that looks like a floor or a wall.

Collinear points Points that lie on the same line

Coplanar points Points that lie in the same plane

Defined Terms Terms that can be described using known words

Line segment, endpoints Part of a line that consists of two points, called endpoints, and all the points on the line between the endpoints

Ray The ray AB consists of the endpoint A and all points on $\overleftrightarrow{AB}$ that lie on the same side of A as B.

Opposite rays If point C lies on $\overleftrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

Intersection The intersection of two or more geometric figures is the set of points that the figures have in common.

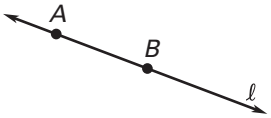
Your Notes

UNDEFINED TERMS

Point A **point** has ___no___ dimension. It is represented by a ___dot___.

point *A*

Line A **line** has ___one___ dimension. It is represented by a ___line___ with two arrowheads, but it extends without end.



line ℓ, line AB ($\overleftrightarrow{AB}$), or line BA ($\overleftrightarrow{BA}$)

Through any ___two___ points, there is exactly ___one___ line. You can use any ___two___ points on a line to name it.

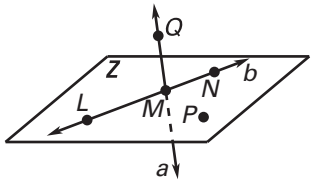
Plane A **plane** has ___two___ dimensions. It is represented by a shape that looks like a floor or wall, but it extends without end.

plane *M* or plane *ABC*

Through any ___three___ points not on the same line, there is exactly ___one___ plane. You can use ___three___ points that are not all on the same line to name a plane.

There is a line through points *L* and *Q* that is not shown in the diagram. Try to imagine what plane *LMQ* would look like if it were shown.

Example 1 ***Name points, lines, and planes***



a. Give two other names for $\overleftrightarrow{LN}$. Give another name for plane *Z*.

b. Name three points that are collinear. Name four points that are coplanar.

a. Other names for $\overleftrightarrow{LN}$ are ___$\overleftrightarrow{LM}$___ and ___line *b*___. Other names for plane *Z* are plane ___*LMP*___ and ___*LNP*___.

b. Points ___*L, M,* and *N*___ lie on the same line, so they are collinear. Points ___*L, M, N,* and *P*___ lie on the same plane, so they are coplanar.

✔ ***Checkpoint*** **Use the diagram in Example 1.**

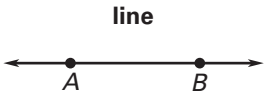
1. Give two other names for $\overleftrightarrow{MQ}$. Name a point that is not coplanar with points *L, N,* and *P*.

$\overleftrightarrow{QM}$ and line *a*; point *Q*

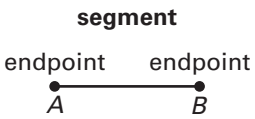
Your Notes

DEFINED TERMS: SEGMENTS AND RAYS

Line *AB* (written as $\overleftrightarrow{AB}$) and points *A* and *B* are used here to define the terms below.

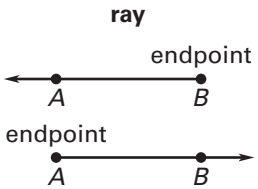


Segment The **line segment** *AB*, or segment *AB*, (written as $\overline{AB}$) consists of the endpoints *A* and *B* and all points on $\overleftrightarrow{AB}$ that are between *A* and *B*.



Note that $\overline{AB}$ can also be named $\overline{BA}$.

Ray The **ray** *AB* (written as $\overrightarrow{AB}$) consists of the endpoint *A* and all points on $\overleftrightarrow{AB}$ that lie on the same side of *A* as *B*.



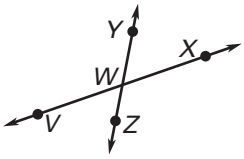
Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.

In Example 2, $\overrightarrow{WY}$ and $\overrightarrow{WX}$ have a common endpoint, but are *not* collinear. So they are not opposite rays.

Example 2 ***Name segments, rays, and opposite rays***

a. Give another name for $\overline{VX}$.

b. Name all rays with endpoints *W*. Which of these rays are opposite rays?



a. Another name for $\overline{VX}$ is $\overline{XV}$.

b. The rays with endpoint *W* are $\overrightarrow{WV}$, $\overrightarrow{WY}$, $\overrightarrow{WX}$, and $\overrightarrow{WZ}$.

The opposite rays with endpoint *W* are $\overrightarrow{WV}$ and $\overrightarrow{WX}$, and $\overrightarrow{WY}$ and $\overrightarrow{WZ}$.

✔ ***Checkpoint*** **Use the diagram in Example 2.**

2. Give another name for $\overline{YW}$.

$\overline{WY}$

3. Are $\overrightarrow{VX}$ and $\overrightarrow{XV}$ the same ray? Are $\overrightarrow{VW}$ and $\overrightarrow{VX}$ the same ray? *Explain.*

No, the rays do not have the same endpoint; Yes, the rays have a common endpoint, are collinear, and consist of the same points.

Your Notes

Sketching a diagram is a way to show geometric relationships informally. A sketch can be made by hand without using measuring tools.

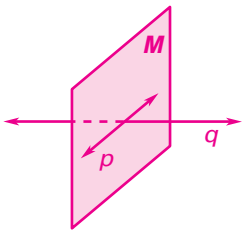
Example 3 ***Sketch intersections of lines and planes***

a. Sketch a plane and a line that intersects the plane at more than one point.

b. Sketch a plane and a line that is in the plane. Sketch another line that intersects the line and plane at a point.

a.

b.



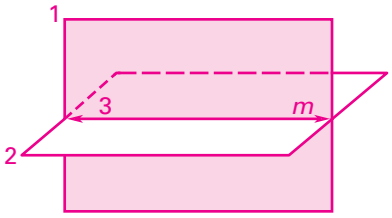
Example 4 ***Sketch intersections of planes***

Sketch two planes that intersect in a line.

Step 1 **Sketch** one plane as if you are facing it.

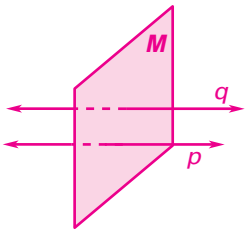
Step 2 **Sketch** a second plane that is horizontal. Use dashed lines to show where one plane is hidden.

Step 3 **Sketch** the line of intersection.



✔ ***Checkpoint*** **Complete the following exercises.**

4. Sketch two different lines that intersect a plane at different points.

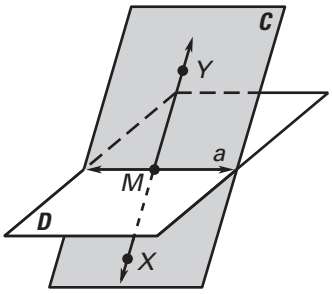


5. Name the intersection of $\overleftrightarrow{MX}$ and line *a*.

point *M*

6. Name the intersection of plane *C* and plane *D*.

line *a*



Homework

1.2 Use Segments and Congruence

Goal • Use segment postulates to identify congruent segments.

Your Notes

VOCABULARY

Postulate, axiom A rule that is accepted without proof

Theorem A rule that can be proved

Coordinate The real number that corresponds to a point

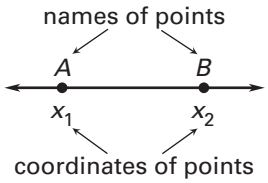
Distance The distance between two points *A* and *B,* written as *AB,* is the absolute value of the difference of the coordinates of *A* and *B*.

Between When three points are collinear, you can say that one point is between the other two.

Congruent segments Line segments that have the same length

POSTULATE 1 RULER POSTULATE

The points on a line can be matched one to one with real numbers. The real number that corresponds to a point is the coordinate of the point.



The distance between points *A* and *B*, written as *AB,* is the absolute value of the difference of the coordinates of *A* and *B*.

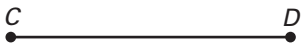
$AB = |x_2 - x_1|$

Your Notes

A ruler is a drawing tool that allows you to measure segment lengths. You can also use a ruler to draw segments of given lengths.

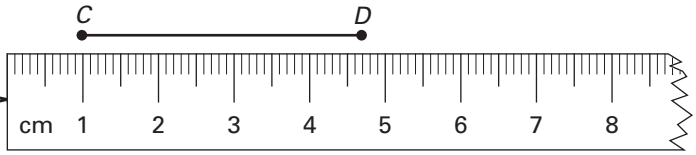
Example 1 ***Apply the Ruler Postulate***

Measure the length of $\overline{CD}$ to the nearest tenth of a centimeter.



Solution

Align one mark of a metric ruler with *C*. Then estimate the coordinate of *D*. For example, if you align *C* with 1, *D* appears to align with 4.7 .



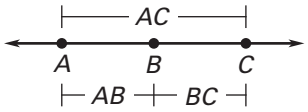
$CD = |\ 4.7 - 1\ | = 3.7$ **Ruler postulate**

The length of $\overline{CD}$ is about 3.7 centimeters.

POSTULATE 2 SEGMENT ADDITION POSTULATE

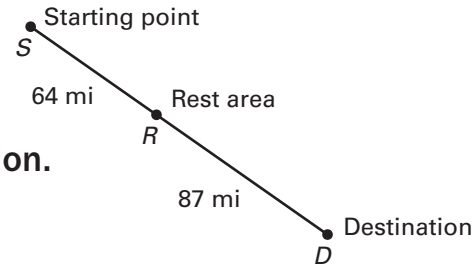
If *B* is between *A* and *C*, then $AB + BC = AC$.

If $AB + BC = AC$, then *B* is between *A* and *C*.



Example 2 ***Apply the Segment Addition Postulate***

Road Trip The locations shown lie in a straight line. Find the distance from the starting point to the destination.



Solution

The rest area lies between the starting point and the destination, so you can apply the Segment Addition Postulate.

$SD = SR + RD$ **Segment Addition Postulate**

$= 64 + 87$ **Substitute for** *SR* **and** *RD* **.**

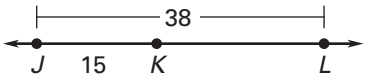
$= 151$ **Add.**

The distance from the starting point to the destination is 151 miles.

Your Notes

Example 3 ***Find a length***

Use the diagram to find *KL*.

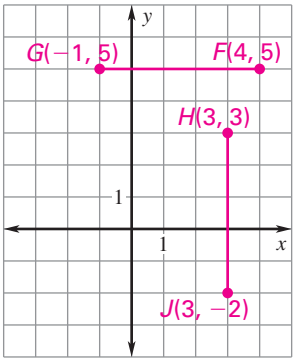


Use the Segment Addition Postulate to write an equation. Then solve the equation to find *KL*.

JL = JK + KL	**Segment Addition Postulate**
38 = 15 + KL	**Substitute for** JL **and** JK **.**
23 = KL	**Subtract** 15 **from each side.**

Example 4 ***Compare segments for congruence***

Plot *F*(4, 5), *G*(−1, 5), *H*(3, 3), and *J*(3, −2) in a coordinate plane. Then determine whether $\overline{FG}$ and $\overline{HJ}$ are congruent.



Horizontal segment: Subtract the x-coordinates of the endpoints.

$FG = |4 - (-1)| = 5$

Vertical segment: Subtract the y-coordinates of the endpoints.

$HJ = |3 - (-2)| = 5$

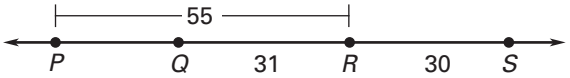
$\overline{FG}$ and $\overline{HJ}$ have the same length. So $\overline{FG} \cong \overline{HJ}$.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Find the length of $\overline{AB}$ to the nearest $\frac{1}{8}$ inch. Then draw a segment with the same length.

$1\frac{7}{8}$ inches; Check drawing

2. Find *QS* and *PQ*.



61; 24

3. Consider the points *A*(−2, −1), *B*(4, −1), *C*(3, 0), and *D*(3, 5). Are $\overline{AB}$ and $\overline{CD}$ congruent? No

Homework

1.3 Use Midpoint and Distance Formulas

Goal • Find lengths of segments in the coordinate plane.

Your Notes

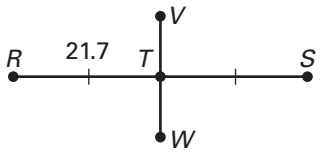
VOCABULARY

Midpoint The point that divides a segment into two congruent segments

Segment bisector A point, ray, line, line segment, or plane that intersects the segment at its midpoint

Example 1 ***Find segments lengths***

Find *RS*.



Solution

Point T is the midpoint of $\overline{RS}$. So, $RT =$ TS $= 21.7$.

$RS =$ RT + TS **Segment Addition Postulate**

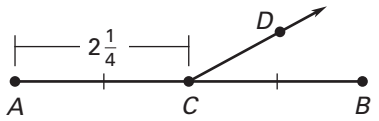
$=$ 21.7 + 21.7 **Substitute.**

$=$ 43.4 **Add.**

The length of $\overline{RS}$ is 43.4 .

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find *AB*.

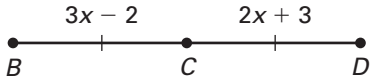


$4\frac{1}{2}$

Your Notes

Example 2 ***Use algebra with segment lengths***

Point C is the midpoint of $\overline{BD}$. Find the length of $\overline{BC}$.



Solution

Step 1 Write and solve an equation.

$BC = CD$	**Write equation.**
$3x - 2 = 2x + 3$	**Substitute.**
$x - 2 = 3$	**Subtract** $2x$ **from each side.**
$x = 5$	**Add** 2 **to each side.**

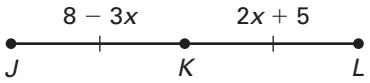
Step 2 Evaluate the expression for BC when $x = 5$.

$$BC = 3x - 2 = 3(5) - 2 = 13$$

So, the length of $\overline{BC}$ is 13 units.

✔ ***Checkpoint*** **Complete the following exercise.**

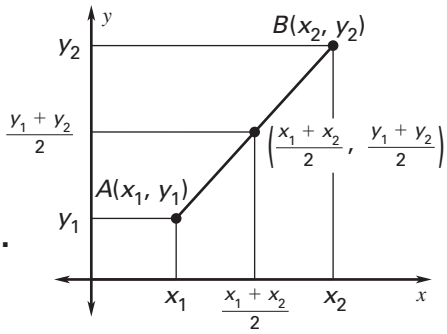
2. Point K is the midpoint of $\overline{JL}$. Find the length of $\overline{KL}$.



$6\frac{1}{5}$

THE MIDPOINT FORMULA

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.



If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Your Notes

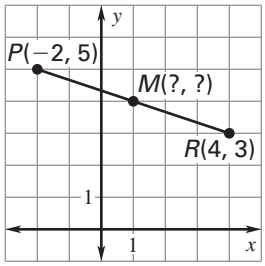
Example 3 ***Use the Midpoint Formula***

a. Find Midpoint The endpoints of $\overline{PR}$ are $P(-2, 5)$ and $R(4, 3)$. Find the coordinates of the midpoint M.

b. Find Endpoint The midpoint of $\overline{AC}$ is $M(3, 4)$. One endpoint is $A(1, 6)$. Find the coordinates of endpoint C.

Solution

a. Use the Midpoint Formula.



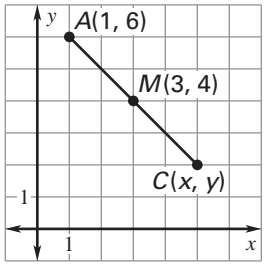
$$M\left(\frac{\boxed{-2} + \boxed{4}}{\boxed{2}}, \frac{\boxed{5} + \boxed{3}}{\boxed{2}}\right)$$

$= M(\underline{1}, \underline{4})$

The coordinates of the midpoint of $\overline{PR}$ are $\underline{M(1, 4)}$.

Multiply each side of the equation by the denominator to clear the fraction.

b. Let (x, y) be the coordinates of endpoint C. Use the Midpoint Formula to find x and y.



Step 1 Find x.

$$\frac{\boxed{1} + x}{2} = \underline{3}$$

$\underline{1} + x = \underline{6}$

$x = \underline{5}$

Step 2 Find y.

$$\frac{\boxed{6} + y}{2} = \underline{4}$$

$\underline{6} + y = \underline{8}$

$y = \underline{2}$

The coordinates of endpoint C are $\underline{(5, 2)}$.

✔ ***Checkpoint*** **Complete the following exercises.**

3. The endpoints of $\overline{CD}$ are $C(-8, -1)$ and $D(2, 4)$. Find the coordinates of the midpoint M.

$M\left(-3, \frac{3}{2}\right)$

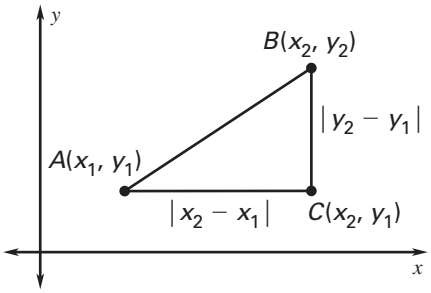
4. The midpoint of $\overline{XZ}$ is $M(5, -6)$. One endpoint is $X(-3, 7)$. Find the coordinates of endpoint Z.

$(13, -19)$

Your Notes

THE DISTANCE FORMULA

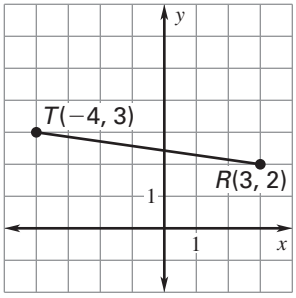
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is



$$AB = \sqrt{(\underline{x_2 - x_1})^2 + (\underline{y_2 - y_1})^2}$$

Example 4 ***Use the Distance Formula***

What is the approximate length of $\overline{RT}$, with endpoints $R(3, 2)$ and $T(-4, 3)$?



Solution

Use the Distance Formula.

$RT = \sqrt{(\underline{x_2 - x_1})^2 + (\underline{y_2 - y_1})^2}$	**Distance Formula**
$= \sqrt{(\underline{-4} - 3)^2 + (\underline{3} - \underline{2})^2}$	**Substitute.**
$= \sqrt{(\underline{-7})^2 + (\underline{1})^2}$	**Subtract.**
$= \sqrt{\underline{49} + \underline{1}}$	**Evaluate powers.**
$= \sqrt{\underline{50}}$	**Add.**
$\approx \underline{7.07}$	**Use a calculator.**

The symbol ≈ means "is approximately equal to."

The length of $\overline{RT}$ is about 7.07 .

✔ ***Checkpoint*** **Complete the following exercise.**

Homework

5. What is the approximate length of $\overline{GH}$, with endpoints $G(5, -1)$ and $H(-3, 6)$?

about 10.63

1.4 Measure and Classify Angles

Goal • Name, measure, and classify angles.

Your Notes

VOCABULARY

Angle An angle consists of two different rays with the same endpoint.

Sides of an angle In an angle, the rays are called the sides of the angle.

Vertex of an angle In an angle, the endpoint is the vertex of the angle.

Measure of an angle In $\angle AOB$, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ can be matched one to one with real numbers from 0 to 180. The measure of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Acute angle An angle that measures between 0° and 90°

Right angle An angle that measures 90°

Obtuse angle An angle that measures between 90° and 180°

Straight angle An angle that measures 180°

Congruent angles Angles with the same measure

Angle bisector A ray that divides an angle into two angles that are congruent

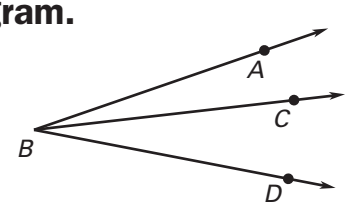
Example 1 ***Name angles***

Name the three angles in the diagram.

$\angle ABC$, or $\angle CBA$

$\angle CBD$, or $\angle DBC$

$\angle ABD$, or $\angle DBA$

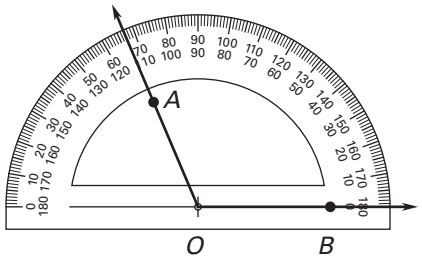


You should not name any of these angles *B* because all three angles have *B* as their vertex.

Your Notes

POSTULATE 3: PROTRACTOR POSTULATE

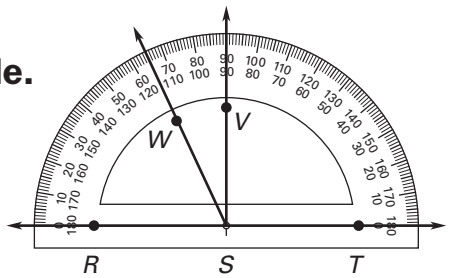
Consider $\overrightarrow{OB}$ and point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180 .



The measure of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Example 2 ***Measure and classify angles***

Use the diagram to find the measure of the indicated angle. Then classify the angle.



a. $\angle WSR$ **b.** $\angle TSW$

c. $\angle RST$ **d.** $\angle VST$

A protractor is a drawing tool that allows you to measure angles. You can also use a protractor to draw angles of given measures.

a. $\overrightarrow{SR}$ is lined up with the 0° on the outer scale of the protractor. $\overrightarrow{SW}$ passes through 65° on the outer scale. So, $m\angle WSR =$ 65° . It is an acute angle.

b. $\overrightarrow{ST}$ is lined up with the 0° on the inner scale of the protractor. $\overrightarrow{SW}$ passes through 115° on the inner scale. So, $m\angle TSW =$ 115° . It is an obtuse angle.

c. $m\angle RST =$ 180° . It is a straight angle.

d. $m\angle VST =$ 90° . It is a right angle.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Name all the angles in the diagram at the right.

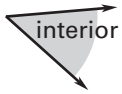
$\angle FGH$ or $\angle HGF$, $\angle FGJ$ or $\angle JGF$, $\angle JGH$ or $\angle HGJ$

2. What type of angles do the x-axis and y-axis form in a coordinate plane?

right angles

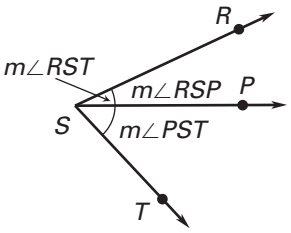
Your Notes

A point is in the *interior* of an angle if it is between points that lie on each side of the angle.



POSTULATE 4: ANGLE ADDITION POSTULATE

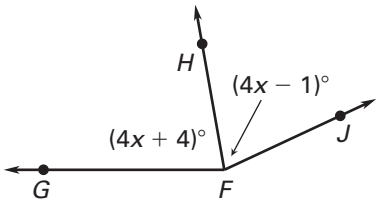
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle$ RSP and $\angle$ PST.



Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle$ RSP $+ m\angle$ PST.

Example 3 ***Find angle measures***

Given that $m\angle GFJ = 155°$, find $m\angle GFH$ and $m\angle HFJ$.



Solution

Step 1 Write and solve an equation to find the value of x.

$m\angle GFJ = m\angle$ GFH $+ m\angle$ HFJ	**Angle Addition Postulate**
155° $= ($ $4x + 4$ $)° + ($ $4x - 1$ $)°$	**Substitute.**
155 $=$ $8x + 3$	**Combine like terms.**
152 $=$ $8x$	**Subtract** 3 **from each side.**
19 $= x$	**Divide each side by** 8 **.**

Step 2 Evaluate the given expressions when $x =$ 19.

$m\angle GFH = ($ $4x + 4$ $)° = ($ $4 \cdot 19 + 4$ $)° =$ 80°.

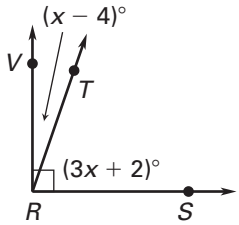
$m\angle HFJ = ($ $4x - 1$ $)° = ($ $4 \cdot 19 - 1$ $)° =$ 75°.

So, $m\angle GFH =$ 80° and $m\angle HFJ =$ 75°.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Given that $\angle VRS$ is a right angle, find $m\angle VRT$ and $\angle TRS$.

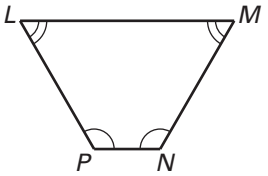
$m\angle VRT = 19°$, $m\angle TRS = 71°$



Your Notes

Example 4 ***Identify congruent angles***

Identify all pairs of congruent angles in the diagram. If $m\angle P = 120°$, what is $m\angle N$?



Solution

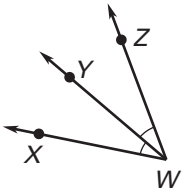
There are two pairs of congruent angles:

$\angle P \cong$ $\angle N$ and $\angle L \cong$ $\angle M$

Because $\angle P \cong$ $\angle N$, $m\angle P =$ $m\angle N$.
So, $m\angle N =$ $120°$.

Example 5 ***Double an angle measure***

In the diagram at the right, $\overrightarrow{WY}$ bisects $\angle XWZ$, and $m\angle XWY = 29°$. Find $m\angle XWZ$.



Solution

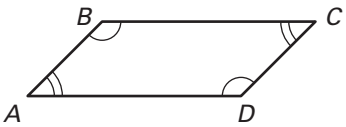
By the Angle Addition Postulate,
$m\angle XWZ =$ $m\angle XWY$ + $m\angle YWZ$.

Because $\overrightarrow{WY}$ bisects $\angle XWZ$, you know
$\angle XWY \cong \angle YWZ$.

So, $m\angle XWY = m\angle YWZ$, and you can write
$m\angle XWZ =$ $m\angle XWY$ + $m\angle YWZ$
$=$ $29°$ + $29°$ $=$ $58°$.

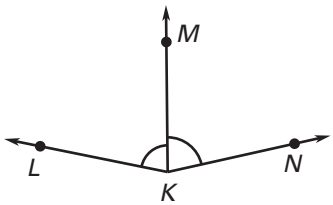
✔ ***Checkpoint*** **Complete the following exercises.**

4. Identify all pairs of congruent angles in the diagram. If $m\angle B = 135°$, what is $m\angle D$?



$\angle B \cong \angle D$ and $\angle A \cong \angle C$; $135°$

5. In the diagram below, $\overrightarrow{KM}$ bisects $\angle LKN$ and $m\angle LKM = 78°$. Find $m\angle LKN$.



$156°$

Homework

1.5 Describe Angle Pair Relationships

Goal • Use special angle relationships to find angle measures.

Your Notes

VOCABULARY

Complementary angles Two angles whose sum is 90°

Supplementary angles Two angles whose sum is 180°

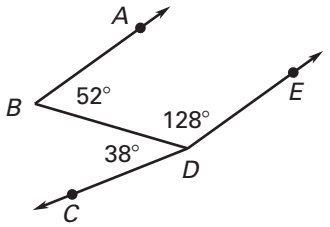
Adjacent angles Two angles that share a common vertex or side, but have no common interior points

Linear pair Two adjacent angles are a linear pair if their noncommon sides are opposite rays.

Vertical angles Two angles are vertical angles if their sides form two pairs of opposite rays.

Example 1 ***Identify complements and supplements***

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because 52° + 38° = 90°, ∠*ABD* and ∠*CDB* are complementary angles.

Because 52° + 128° = 180°, ∠*ABD* and ∠*EDB* are supplementary angles.

Because ∠*CDB* and ∠*BDE* share a common vertex and side, they are adjacent angles.

In Example 1, ∠*BDE* and ∠*CDE* share a common vertex. But they share common interior points, so they are *not* adjacent angles.

Your Notes

Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

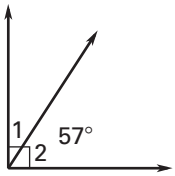
Example 2 ***Find measures of complements and supplements***

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 57°$, find $m\angle 1$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 41°$, find $m\angle 3$.

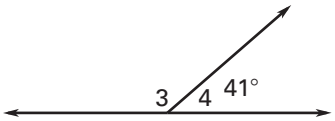
Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.



$m\angle 1 =$ 90° $-$ $m\angle 2$ $=$ 90° $-$ 57° $=$ 33°

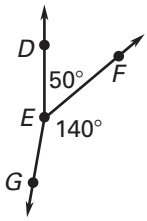
b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

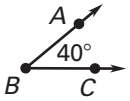


$m\angle 3 =$ 180° $-$ $m\angle 4$ $=$ 180° $-$ 41° $=$ 139°

✔ ***Checkpoint*** **Complete the following exercises.**

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.





complementary: $\angle DEF$ and $\angle ABC$; supplementary; $\angle FEG$ and $\angle ABC$; adjacent: $\angle DEF$ and $\angle FEG$

2. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 73°$, find $m\angle 2$.

17°

3. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 37°$, find $m\angle 3$.

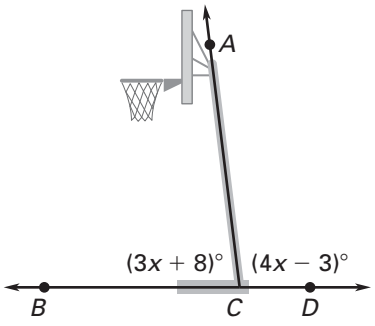
143°

Your Notes

In a diagram, you can assume that a line that looks straight *is* straight. In Example 3, B, C, and D lie on $\overleftrightarrow{BD}$. So, $\angle BCD$ is a straight angle.

Example 3 ***Find angle measures***

Basketball The basketball pole forms a pair of supplementary angles with the ground. Find $m\angle BCA$ and $m\angle DCA$.



Solution

Step 1 **Use** the fact that 180° is the sum of the measures of supplementary angles.

$m\angle BCA + m\angle DCA =$	180°	**Write equation.**
$(3x + 8)° + (4x - 3)° =$	180°	**Substitute.**
$7x + 5 =$	180	**Combine like terms.**
$7x =$	175	**Subtract.**
$x =$	25	**Divide.**

Step 2 **Evaluate** the original expressions when $x =$ 25.

$m\angle BCA = (3x + 8)° = (3 \cdot 25 + 8)° =$ 83°.

$m\angle DCA = (4x - 3)° = (4 \cdot 25 - 3)° =$ 97°.

The angle measures are 83° and 97°.

✔ ***Checkpoint*** **Complete the following exercise.**

4. In Example 3, suppose the angle measures are $(5x + 1)°$ and $(6x + 3)°$. Find $m\angle BCA$ and $m\angle DCA$.

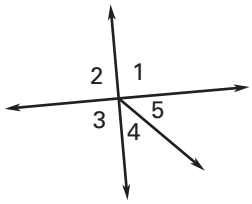
81° and 99°

Your Notes

In the diagram, one side of ∠1 and one side of ∠4 are opposite rays. But the angles are not a linear pair because they are not adjacent.

Example 4 ***Identify angle pairs***

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

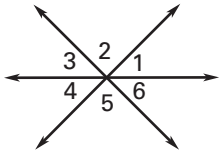
∠1 and ∠3 are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

∠1 and ∠2 are a linear pair. ∠2 and ∠3 are a linear pair.

✔ ***Checkpoint*** **Complete the following exercise.**

5. Identify all of the linear pairs and all of the vertical angles in the figure.



linear pairs: none; vertical angles: ∠1 and ∠4, ∠2 and ∠5, ∠3 and ∠6

Example 5 ***Find angle measures in a linear pair***

Two angles form a linear pair. The measure of one angle is 4 times the measure of the other. Find the measure of each angle.

Solution

Let $x°$ be the measure of one angle. The measure of the other angle is $4x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

$x° + 4x° = 180°$ **Write an equation.**

$5x = 180$ **Combine like terms.**

$x = 36$ **Divide each side by 5.**

The measures of the angles are 36° and 4(36°) = 144°.

One angle measure should be four times the other angle measure. The measures should also have a sum of 180° for the angles to form a linear pair.

Your Notes

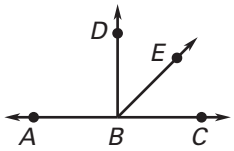
✔ ***Checkpoint*** **Complete the following exercise.**

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the meaure of each angle.

45° and 135°

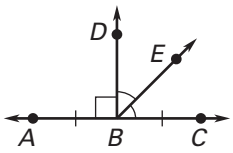
CONCEPT SUMMARY: INTERPRETING A DIAGRAM

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right.



- All points shown are coplanar.
- Points *A*, *B*, and *C* are collinear, and *B* is between *A* and *C*.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point *B*.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the interior of $\angle DBC$.

In the diagram above, you cannot conclude that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



Homework

1.6 Classify Polygons

Goal • Classify polygons.

Your Notes

VOCABULARY

Polygon A polygon is a closed plane figure with the following properties: (1) It is formed by three or more line segments called sides. (2) Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Sides The sides of a polygon are the line segments that form the polygon.

Vertex A vertex of a polygon is an endpoint of a side of the polygon.

Convex A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

Concave A concave polygon is a polygon that is not convex.

***n*-gon** An *n*-gon is a polygon with *n* sides.

Equilateral A polygon is equilateral if all of its sides are congruent.

Equiangular A polygon is equiangular if all of its angles in the interior are congruent.

Regular A polygon is regular if all sides and all angles are congruent.

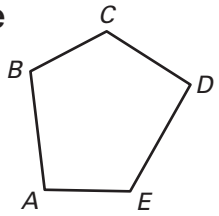
Your Notes

IDENTIFYING POLYGONS

In geometry, a figure that lies in a plane is called a *plane figure*. A polygon is a closed plane figure with the following properties.

1. It is formed by three or more line segments called sides.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

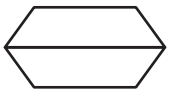
Each endpoint of a side is a vertex of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, *ABCDE* and *CDEAB* are both correct names for the polygon at the right.

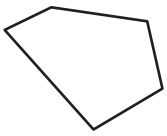


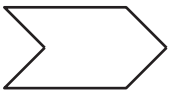
A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

Example 1 ***Identify polygons***

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

a. 

b. 

c. 

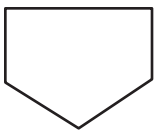
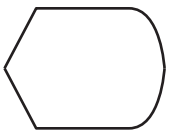
Solution

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. The figure is a concave polygon.

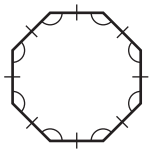
✔ ***Checkpoint*** **Tell whether the figure is a polygon and whether it is *convex* or *concave*.**

1.  convex polygon	**2.**  not a polygon

Your Notes

Example 2 ***Classify polygons***

Classify the polygon by the number of sides. Tell which terms apply to the polygon: *equilateral*, *equiangular*, *regular*, or *not regular*.

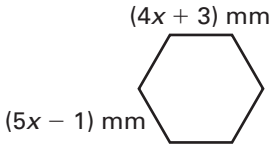


Solution

The polygon has 8 sides. It is equilateral and equiangular, so it is a regular octagon.

Example 3 ***Find side lengths***

The head of a bolt is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal bolt. Find the length of a side.



Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$4x + 3 = 5x - 1$ **Write an equation.**

$4 = x$ **Simplify.**

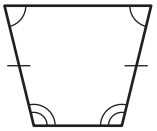
Then evaluate one of the expressions to find a side length when $x = 4$. $4x + 3 = 4(4) + 3 = 19$

The length of a side is 19 millimeters.

Check that this is a reasonable side length for a hexagonal bolt.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Classify the polygon by the number of sides. Tell which terms apply to the polygon: *equilateral*, *equiangular*, ***regular*, or *not regular*.**



quadrilateral; not regular

4. The expressions $(4x + 8)°$ and $(5x - 5)°$ represent the measures of two of the congruent angles in Example 3. Find the measure of an angle.

60°

Homework

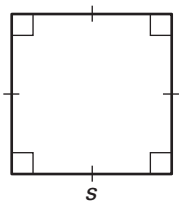
1.7 Find Perimeter, Circumference, and Area

Goal • Find dimensions of polygons.

Your Notes

FORMULAS FOR PERIMETER *P*, AREA *A*, AND CIRCUMFERENCE *C*

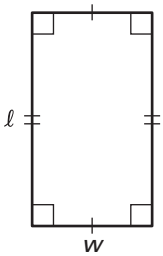
Square
side length s



$P =$ $4s$

$A =$ s^2

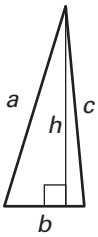
Rectangle
length ℓ and width w



$P =$ $2\ell + 2w$

$A =$ ℓw

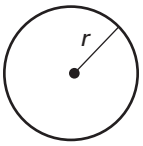
Triangle
side lengths a, b, and c, base b, and height h.



$P =$ $a + b + c$

$A =$ $\frac{1}{2}bh$

Circle
radius r



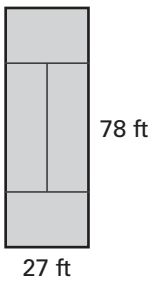
$C =$ $2\pi r$

$A =$ πr^2

Pi (π) is the ratio of a circle's circumference to its diameter.

Example 1 ***Find the perimeter and area of a rectangle***

Tennis The in-bounds portion of a singles tennis court is shown. Find its perimeter and area.



Perimeter

$P = 2\ell + 2w$

$= 2(\underline{78}) + 2(\underline{27})$

$= \underline{210}$

Area

$A = \ell w$

$= \underline{78}(\underline{27})$

$= \underline{2106}$

The perimeter is 210 ft and the area is 2106 ft^2.

✔ ***Checkpoint*** **Complete the following exercise.**

1. In Example 1, the width of the in-bounds rectangle increases to 36 feet for doubles play. Find the perimeter and area of the in-bounds rectangle.

perimeter: 228 ft, area: 2808 ft^2

Your Notes

The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π. Unless told otherwise, use 3.14 for π.

Example 2 ***Find the circumference and area of a circle***

Archery The smallest circle on an Olympic target is 12 centimeters in diameter. Find the approximate circumference and area of the smallest circle.

Solution

First find the radius. The diameter is 12 centimeters, so the radius is $\frac{1}{2}$(12) = 6 centimeters.

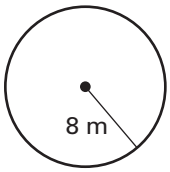
Then find the circumference and area. Use 3.14 for π.

$P = 2\pi r \approx 2($ 3.14 $)($ 6 $) =$ 37.68 cm

$A = \pi r^2 \approx$ 3.14 (6 $)^2 =$ 113.04 cm^2

✔ ***Checkpoint*** **Find the approximate circumference and area of the circle.**

2.

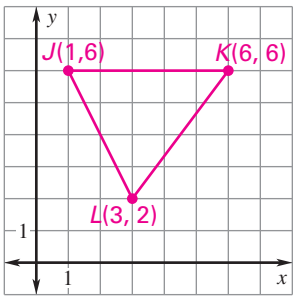


$C \approx 50.24$ m; $A \approx 200.96$ m^2

Example 3 ***Using a coordinate plane***

Triangle *JKL* has vertices *J*(1, 6), *K*(6, 6), and *L*(3, 2). Find the approximate perimeter of triangle *JKL*.

Solution



First draw triangle *JKL* in a coordinate plane. Then find the side lengths. Because $\overline{JK}$ is horizontal, use the Ruler Postulate to find *JK*. Use the Distance Formula to find *JL* and *LK*.

Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

$JK = |$ 6 $-$ 1 $| =$ 5 units

$JL = \sqrt{(3 - 1)^2 + (2 - 6)^2} = \sqrt{20} \approx$ 4.5 units

$LK = \sqrt{(6 - 3)^2 + (6 - 2)^2} = \sqrt{25} =$ 5 units

Then find the perimeter.

$P = JK + JL + LK \approx$ 5 + 4.5 + 5 = 14.5 units.

Your Notes

Example 4 ***Solve a multi-step problem***

Lawn care You are using a roller to smooth a lawn. You can roll about 125 square yards in one minute. About how many minutes does it take to roll a lawn that is 120 feet long and 75 feet wide?

Solution

You can roll the lawn at a rate of 125 square yards per minute. So, the amount of time it takes you to roll the lawn depends on its area.

Step 1 **Find** the area of the rectangular lawn.

Area $= \ell w =$ 120 (75) $=$ 9000 ft^2

The rolling rate is in square yards per minute. Rewrite the area of the lawn in square yards. There are 3 feet in 1 yard, and $3^2 =$ 9 square feet in one square yard.

$$9000\ \cancel{ft^2} \cdot \frac{1\ yd^2}{\boxed{9}\ \cancel{ft^2}} = 1000\ yd^2$$ **Use unit analysis.**

Step 2 **Write** a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let t represent the total time (in minutes) needed to roll the lawn.

Area of lawn (yd^2)	=	Rolling rate (yd^2 per min)	×	Total time (min)

$1000 = 125 \cdot t$ **Substitute.**

$8 = t$ **Divide each side by** 125 **.**

It takes about 8 minutes to roll the lawn.

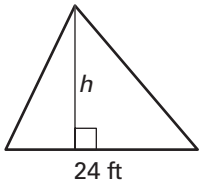
You can roll about 1250 yards in 10 minutes. Use this fact to check that your solution is reasonable for the lawn area you found in Step 1.

Your Notes

In Example 5, you may want to make and label a sketch of the triangle. The sketch will not be exact, but it will help you visualize the given information.

Example 5 ***Find unknown length***

The base of a triangle is 24 feet. Its area is 216 square feet. Find the height of the triangle.



Solution

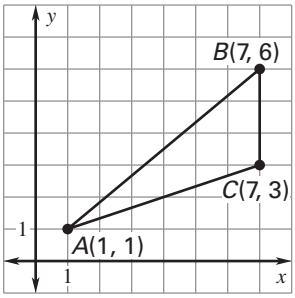
$A = \frac{1}{2}bh$		**Area of a triangle**
216	$= \frac{1}{2}(24)(h)$	**Substitute.**
216	$= 12h$	**Multiply.**
18	$= h$	**Solve for *h*.**

The height is 18 feet.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Find the perimeter of the triangle shown at the right.

about 17.1 units



4. Suppose a lawn is half as long and half as wide as the lawn in Example 4. Will it take half the time to roll the lawn? *Explain.*

No, it will take a quarter of the time to roll the lawn because it is a quarter of the original area.

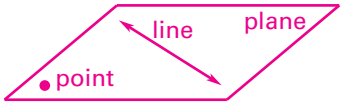
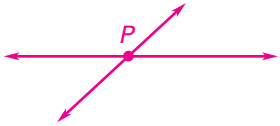
Homework

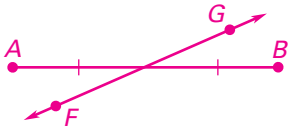
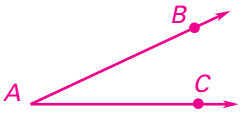
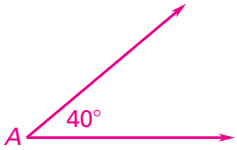
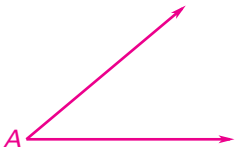
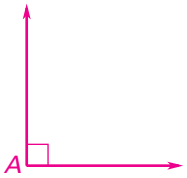
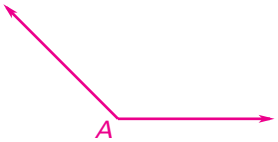
5. The area of a triangle is 96 square inches, and its height is 8 inches. Find the length of its base.

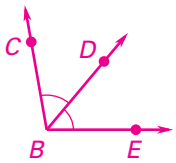
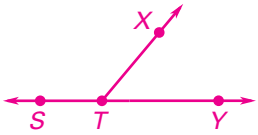
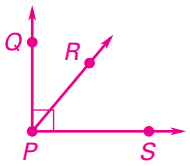
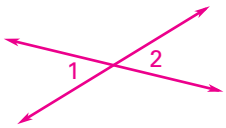
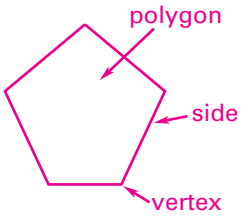
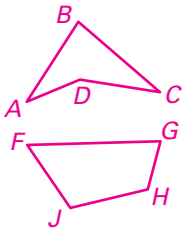
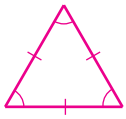
24 inches

Words to Review

Give an example of the vocabulary word.

Point, line, plane  	**Collinear points**  A and B are collinear points.
Coplanar points  D and T are coplanar points.	**Line segment, endpoints**  $\overline{CD}$ is a line segment with endpoints C and D.
Ray  $\overrightarrow{XY}$ is a ray with initial point X.	**Opposite rays**  If C is between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.
Intersection  The intersection of two different lines is a point.	**Postulate, axiom** A postulate, or axiom, is a rule that is accepted without proof.
Coordinate  The coordinates of points A and B are x_1 and x_2.	**Distance**  $AB = \lvert x_2 - x_1 \rvert$ The distance between points A and B is $\lvert x_2 - x_1 \rvert$.

Between  Point C is between Points A and B.	**Congruent segments**  $\overline{AB}$ and $\overline{CD}$ are congruent.
Midpoint  M is the midpoint of $\overline{AB}$.	**Segment bisector**  $\overleftrightarrow{FG}$ is a segment bisector of $\overline{AB}$.
Angle, sides, vertex  Sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form $\angle A$. The vertex is A.	**Measure of an angle**  The measure of $\angle A$ is 40°.
Acute angle  $0° < m\angle A < 90°$	**Right angle**  $m\angle A = 90°$
Obtuse angle  $90° < m\angle A < 180°$	**Straight angle**  $m\angle A = 180°$

Angle bisector, congruent angles  $\overrightarrow{BD}$ is an angle bisector of $\angle CBE$. $\angle CBD$ and $\angle DBE$ are congruent.	**Supplementary angles, linear pair**  $\angle STX$ and $\angle XTY$ are supplementary. $\angle STX$ and $\angle XTY$ are a linear pair.
Complementary angles, adjacent angles  $\angle QPR$ and $\angle RPS$ are complementary. $\angle QPR$ and $\angle RPS$ are adjacent.	**Vertical angles**  $\angle 1$ and $\angle 2$ are vertical angles.
Polygon, side, vertex  	**Concave, convex** *ABCD* is concave. *FGHJ* is convex. 
***n*-gon** An *n*-gon is a polygon with *n* sides.	**Equilateral, equiangular, regular**  The polygon is equilateral and equiangular, so it is regular.

Review your notes and Chapter 1 by using the Chapter Review on pages 61–64 of your textbook.

2.1 Use Inductive Reasoning

Goal • Describe patterns and use inductive reasoning.

Your Notes

VOCABULARY

Conjecture A conjecture is an unproven statement that is based on observations.

Inductive Reasoning Inductive reasoning is the process of finding a pattern for specific cases and then writing a conjecture for the general case.

Counterexample A counterexample is a specific case for which the conjecture is false.

Example 1 ***Describe a visual pattern***

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1 

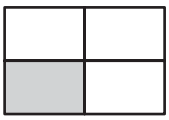
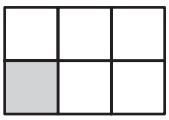
Figure 2 

Figure 3 

Solution

Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eighths. Shade the section just below the horizontal segment at the left.

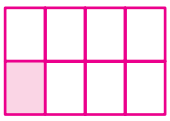


Figure 4

✔ ***Checkpoint*** **Complete the following exercise.**

1. Sketch the fifth figure in the pattern in Example 1.

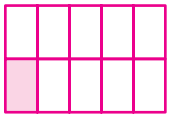


Figure 5

Your Notes

Three dots (. . .) tell you that the pattern continues.

Example 2 ***Describe the number pattern***

Describe the pattern in the numbers −1, −4, −16, −64, ... Write the next three numbers in the pattern.

Notice that each number in the pattern is four times the previous number.

−1, −4, −16, −64, . . .

× 4 × 4 × 4 × 4

The next three numbers are −256, −1024, and −4096.

Example 3 ***Make a conjecture***

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?

+ 1 + 2 + 3 + ?

Conjecture You can connect five noncollinear points 6 + 4, or 10 different ways.

Checkpoint **Complete the following exercises.**

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

The numbers are increasing by 1.5; 7, 8.5, 10.

3. Rework Example 3 if you are given six noncollinear points.

15 different ways

Your Notes

When you test a conjecture and find support for it, you are not proving it. In Example 4, a proof for all cases would use algebra. Let n be any odd integer,

$n + \underline{(n + 2)} + (n + 4)$

$= \underline{3n + 6}$

$= \underline{3(n + 2)}$

Example 4 ***Make and test a conjecture***

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 Find a pattern using groups of small numbers.

$1 + 3 + 5 = \underline{9}$	$3 + 5 + 7 = \underline{15}$
$= 3 \cdot 3$	$= \underline{5} \cdot 3$
$5 + 7 + 9 = \underline{21}$	$7 + 9 + 11 = \underline{27}$
$= \underline{7} \cdot 3$	$= \underline{9} \cdot 3$

Conjecture The sum of any three consecutive odd numbers is three times <u>the second number</u>.

Step 2 Test your conjecture using other numbers.

$-1 + 1 + 3 = \underline{3} = \underline{1} \cdot 3$ ✓

$103 + 105 + 107 = \underline{315} = \underline{105} \cdot 3$ ✓

✔ ***Checkpoint*** **Complete the following exercise.**

4. Make and test a conjecture about the sign of the product of any four negative numbers.

The result of the product of four negative numbers is a positive number; $(-1)(-2)(-5)(-1) = 10$.

Example 5 ***Find a counterexample***

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

To find a counterexample, you need to find a difference that is <u>greater</u> than the <u>larger</u> number.

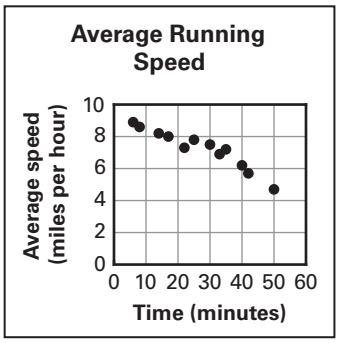
$8 - (-4) = \underline{12}$

Because $\underline{12} \not< \underline{8}$, a counterexample exists. The conjecture is false.

Your Notes

Example 6 ***Making conjectures from data displays***

The scatter plot shows the average running speed of a runner over various time intervals. Make a conjecture based on the graph.



Solution

The scatter plot shows that the average speeds decrease as the time increases. So, one possible conjecture is that the average running speed over a 60-minute interval will be slower than over a 10-minute time interval.

Checkpoint **Complete the following exercises.**

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

$$\frac{4}{\frac{1}{2}} = 8$$

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

Sample answer: The average speed of the runner over a 60-minute interval will be less than 5 miles per hour; The average speed was less than 5 miles per hour over 50 minutes, and overall, the speeds decrease as the times increase.

Homework

2.2 Analyze Conditional Statements

Goal • Write definitions as conditional statements.

Your Notes

VOCABULARY

Conditional statement A logical statement that has two parts, a hypothesis and a conclusion.

If-then form A form of a conditional statement in which the "if" part contains the hypothesis and the "then" part contains the conclusion.

Hypothesis A hypothesis is the "if" part of a conditional statement.

Conclusion A conclusion is the "then" part of a conditional statement.

Negation The negation of a statement is the opposite of the original statement.

Converse The converse of a conditional statement is formed by switching the hypothesis and conclusion.

Inverse The inverse of a conditional statement is formed by negating both the hypothesis and conclusion.

Contrapositive The contrapositive of a conditional statement is formed by writing the converse and then negating both the hypothesis and conclusion.

Equivalent statements Equivalent statements are two statements that are both true or both false.

Perpendicular lines Two lines that intersect to form a right angle are perpendicular lines.

Biconditional statement A statement that contains the phrase "if and only if."

Your Notes

Example 1 ***Rewrite a statement in if-then form***

Write an equivalent conditional statement in if-then form.

All vertebrates have a backbone.

Solution

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If an animal is a vertebrate, then it has a backbone.

✔ ***Checkpoint*** **Write an equivalent conditional statement in if-then form.**

1. All triangles have 3 sides. If a figure is a triangle, then it has 3 sides.	**2.** When $x = 2$, $x^2 = 4$. If $x = 2$, then $x^2 = 4$.

Example 2 ***Write four related conditional statements***

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are an Olympian, then you are an athlete. *True*, Olympians are athletes.

Converse If you are an athlete, then you are an Olympian. *False*, not all athletes are Olympians.

Inverse If you are not an Olympian, then you are not an athlete. *False*, even if you are not an Olympian, you can still be an athlete.

Contrapositive If you are not an athlete, then you are not an Olympian. *True*, a person who is not an athlete cannot be an Olympian.

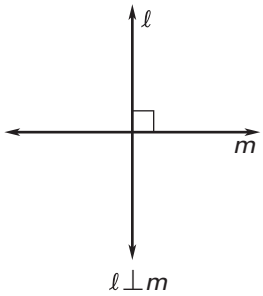
Your Notes

PERPENDICULAR LINES

Definition If two lines intersect to form a right angle, then they are perpendicular lines.

The definition can also be written using the converse: If any two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.

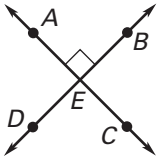


Example 3 *Use definitions*

Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.

a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AED$ and $\angle BEC$ are a linear pair.



Solution

a. The statement is *true*. The right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. The statement is *false*. Because $\angle AED$ and $\angle BEC$ are not adjacent angles, $\angle AED$ and $\angle BEC$ are not a linear pair.

Example 4 *Write a biconditional*

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution

Converse: If two lines are parallel, then they lie in the same plane and do not intersect.

Biconditional: Two lines are parallel if and only if they lie in the same plane and do not intersect.

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Squares are rectangles." Decide whether each statement is *true* or *false*.

If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.

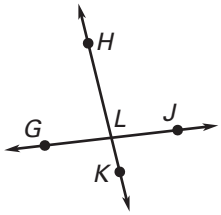
Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.

Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.

Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.

---

4. Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.



a. $\angle GLK$ and $\angle JLK$ are supplementary.

b. $\overleftrightarrow{GJ} \perp \overleftrightarrow{HK}$

(a) True; linear pairs of angles are supplementary.

(b) False; it is not known that the lines intersect at right angles.

---

5. Write the statement below as an equivalent biconditional.

Statement: If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.

A student is in group A if and only if the student is a boy.

Homework

2.3 Apply Deductive Reasoning

Goal • Form logical arguments using deductive reasoning.

Your Notes

VOCABULARY

Deductive Reasoning Using facts, definitions, accepted properties, and the laws of logic to form a logical argument

The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

LAWS OF LOGIC

Law of Detachment If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.
If hypothesis p, then conclusion r.

If these statements are true, then this statement is true.

Example 1 ***Use the Law of Detachment***

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

b. Jesse goes to the gym every weekday. Today is Monday.

Solution

a. Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is a weekday," and the conclusion is "then Jesse goes to the gym."

"Today is Monday" satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.

Your Notes

Example 2 ***Use the Law of Syllogism***

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

b. If $x^2 > 36$, then $x^2 > 30$. If $x > 6$, then $x^2 > 36$.

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

The order in which the statements are given does not affect whether you can use the Law of Syllogism.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.

If Ron eats lunch today, then he will drink a glass of milk.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If $x > 6$, then $x^2 > 30$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

Checkpoint **Complete the following exercises.**

1. If $0° < m\angle A < 90°$, then A is acute. The measure of $\angle A$ is 38°. Using the Law of Detachment, what statement can you make?

$\angle A$ is acute.

2. State the law of logic that is illustrated below.

If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

If you do your homework, then you can watch your favorite show.

Law of Syllogism

Your Notes

Example 3 ***Use inductive and deductive reasoning***

What conclusion can you make about the sum of an odd integer and an odd integer?

Solution

Step 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$-3 + 5 =$ 2 , $-1 + 5 =$ 4 , $3 + 5 =$ 8

$-3 + (-5) =$ −8 , $1 + (-5) =$ −4 ,
$3 + (-5) =$ −2

Conjecture: Odd integer + Odd integer = Even integer

The specific examples listed in Step 1 *support* the conjecture but do not *prove* it. A proof requires a general argument as in Step 2.

Step 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ and $2m$ are even integers because any integer multiplied by 2 is even .

$2n -$ 1 and $2m +$ 1 are odd integers because $2n$ and $2m$ are even integers.

$(2n -$ 1 $) + (2m +$ 1 $)$ represents the sum of an odd integer $2n -$ 1 and an odd integer $2m +$ 1 .

$(2n -$ 1 $) + (2m +$ 1 $) =$ 2 $(n + m)$

The result is the product of 2 and an integer $n + m$. So, 2 $(n + m)$ is an even integer.

The sum of an odd integer and an odd integer is an even integer.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

The sum of a negative integer and itself is twice the integer; $-n + (-n) = -2n = 2(-n)$.

Your Notes

Example 4 ***Compare inductive and deductive reasoning***

Decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. Explain your reasoning.

a. Each time Eric mows his lawn, it takes longer than 30 minutes. So the next time Eric mows his lawn, it will take longer than 30 minutes.

b. Birds are the only animals that have feathers. A penguin has feathers. So a penguin is a type of bird.

Solution

a. Inductive reasoning, because a pattern is used to reach the conclusion.

b. Deductive reasoning, because facts about animals and the laws of logic are used to reach a conclusion.

Checkpoint **Complete the following exercises.**

4. Use inductive reasoning to write another statement about the graph in Example 4.

Sample answer: The faster the average speed of the runner, the less time he or she is running.

5. Give an example of when you used deductive reasoning in everyday life.

Check examples.

Homework

Focus On Reasoning

Use after Lesson 2.3

Symbolic Notation and Truth Tables

Goal • Use symbolic notation to represent logical statements.

Your Notes

VOCABULARY

Truth value Description of a statement as either true (T) or false (F)

Truth table Table that shows the truth values of a conditional statement

Logically equivalent Relationship describing two statements that have the same truth value

SYMBOLIC NOTATION

Let p be "two angles are congruent" and let q be "the measures of the angles are equal."

Conditional If p, then q. $p \rightarrow q$

If two angles are congruent, then their measures are equal.

Converse If q, then p. $q \rightarrow p$

If the measures of two angles are equal, then they are congruent.

Inverse If not p, then not q. $\sim p \rightarrow \sim q$

If two angles are not congruent, then their measures are not equal.

Contrapositive If not q, then not p. $\sim q \rightarrow \sim p$

If the measures of two angles are not equal, then they are not congruent.

Biconditional p if and only if q. $p \leftrightarrow q$

Two angles are congruent if and only if their measures are equal.

Your Notes

Example 1 *Use symbolic notation*

Let p be "the sky is dark" and let q be "it is nighttime."

a. Write the conditional statement $p \rightarrow q$ in words.

b. Write the converse $q \rightarrow p$ in words.

c. Write the biconditional $p \rightarrow q$ in words.

Solution

a. If the sky is dark, then it is nighttime.

b. If it is nighttime, then the sky is dark.

c. The sky is dark if and only if it is nighttime.

Example 2 *Make a truth table*

For a conditional statement with hypothesis p and conclusion q, make a truth table for $\sim q \rightarrow p$.

p	q	$\sim q$	$\sim q \rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

Checkpoint **Complete the following exercises.**

1. Write the inverse of the conditional statement, "If two angles are complementary, then their measures add up to 90°."

If two angles are not complementary, then their measures do not add up to 90°.

2. For a conditional statement with hypothesis p and conclusion q, make a truth table for $\sim q \rightarrow \sim p$.

p	q	$\sim q$	$\sim p$	$\sim q \rightarrow p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Homework

2.4 Use Postulates and Diagrams

Goal • Use postulates involving points, lines, and planes.

Your Notes

VOCABULARY

Line perpendicular to a plane A line is perpendicular to a plane if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

POINT, LINE, AND PLANE POSTULATES

Postulate 5 Through any two points there exists exactly one line.

Postulate 6 A line contains at least two points.

Postulate 7 If two lines intersect, then their intersection is exactly one point.

Postulate 8 Through any three noncollinear points there exists exactly one plane.

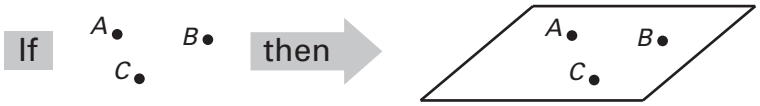
Postulate 9 A plane contains at least three noncollinear points.

Postulate 10 If two points lie in a plane, then the line containing them lies in the plane.

Postulate 11 If two planes intersect, then their intersection is a line.

Example 1 ***Identify a postulate illustrated by a diagram***

State the postulate illustrated by the diagram.



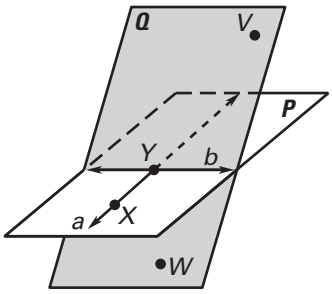
Solution

Postulate 8 Through any three noncollinear points there exists exactly one plane.

Your Notes

Example 2 ***Identify postulates from a diagram***

Use the diagram to write examples of Postulates 9 and 11.



Postulate 9 Plane Q contains at least three noncollinear points, W, V, and Y.

Postulate 11 The intersection of plane *P* and plane *Q* is line *b*.

✔ ***Checkpoint*** **Use the diagram in Example 2 to complete the following exercises.**

1. Which postulate allows you to say that the intersection of line *a* and line *b* is a point?

Postulate 7

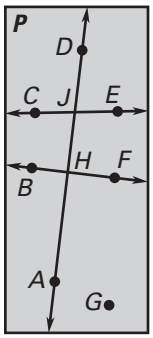
2. Write examples of Postulates 5 and 6.

Line *a* passes through *X* and *Y*; line *a* contains points *X* and *Y*.

CONCEPT SUMMARY: INTERPRETING A DIAGRAM

When you interpret a diagram, you can only assume information about size or measure if it is marked.

YOU CAN ASSUME



All points shown are coplanar.

$\angle AHB$ and $\angle BHD$ are a linear pair.

$\angle AHF$ and $\angle BHD$ are vertical angles.

A, *H*, *J*, and *D* are collinear.

$\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at *H*.

YOU CANNOT ASSUME

G, *F*, and *E* are collinear.

$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect.

$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect.

$\angle BHA \cong \angle CJA$

$\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90°$

Your Notes

Example 3 ***Use given information to sketch a diagram***

Sketch a diagram showing $\overline{RS}$ perpendicular to $\overleftrightarrow{TV}$, intersecting at point *X*.

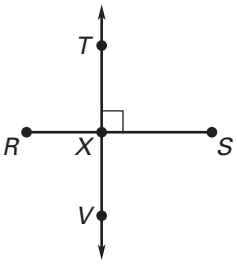
Notice that the picture was drawn so that *X* does not look like a midpoint of $\overline{RS}$.

Solution

Step 1 **Draw** $\overline{RS}$ and label points *R* and *S*.

Step 2 **Draw** a point *X* between *R* and *S*.

Step 3 **Draw** $\overleftrightarrow{TV}$ through *X* so that it is perpendicular to $\overline{RS}$.



Example 4 ***Interpret a diagram in three dimensions***

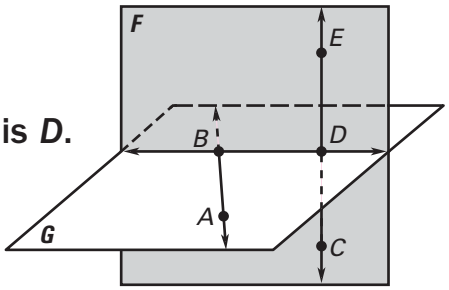
Which of the following statements *cannot* be assumed from the diagram?

E, *D*, and *C* are collinear.

The intersection of $\overleftrightarrow{BD}$ and $\overleftrightarrow{EC}$ is *D*.

$\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$

$\overleftrightarrow{EC} \perp$ plane *G*



Solution

With no right angles marked, you cannot assume that $\overleftrightarrow{BD} \perp \overleftrightarrow{EC}$ or $\overleftrightarrow{EC} \perp$ plane *G*.

✔ ***Checkpoint*** **Complete the following exercises.**

3. In Example 3, if the given information indicated that $\overline{RX}$ and $\overline{XS}$ are congruent, how would the diagram change?

Point *X* would be drawn as the midpoint of $\overline{RS}$ and the congruent segments would be marked.

4. In the diagram for Example 4, can you assume that $\overleftrightarrow{BD}$ is the intersection of plane *F* and plane *G*?

Yes

Homework

2.5 Reason Using Properties from Algebra

Goal • Use algebraic properties in logical arguments.

Your Notes

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then *a* can be substituted for *b* in any equation or expression.

Example 1 ***Write reasons for each step***

Solve $2x + 3 = 9 - x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 3 = 9 - x$	Write original equation.	**Given**
$2x + 3 + x = 9 - x + x$	Add x to each side.	Addition Property of Equality
$3x + 3 = 9$	Combine like terms.	Simplify.
$3x = 6$	Subtract 3 from each side.	Subtraction Property of Equality
$x = 2$	Divide each side by 3.	Division Property of Equality

The value of x is 2.

Your Notes

DISTRIBUTIVE PROPERTY

$a(b + c) =$ $ab + ac$, where a, b, and c are real numbers.

Example 2 ***Use the Distributive Property***

Solve $-4(6x + 2) = 64$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(6x + 2) = 64$	Write original equation.	Given
$-24x - 8 = 64$	Multiply.	Distributive Property
$-24x = 72$	Add 8 to each side.	Addition Property of Equality
$x = -3$	Divide each side by -24.	Division Property of Equality

✔ ***Checkpoint*** **Complete the following exercises.**

1. Solve $x - 5 = 7 + 2x$. Write a reason for each step.

$x - 5 = 7 + 2x$	Given
$x - 5 - x = 7 + 2x - x$	Subtraction Property of Equality
$-5 = 7 + x$	Simplify.
$-12 = x$	Subtraction Property of Equality

2. Solve $4(5 - x) = -2x$. Write a reason for each step.

$4(5 - x) = -2x$	Given
$20 - 4x = -2x$	Distributive Property
$20 = 2x$	Addition Property of Equality
$10 = x$	Division Property of Equality

Your Notes

Example 3 ***Use properties in the real world***

Speed A motorist travels 5 miles per hour slower than the speed limit s for 3.5 hours. The distance traveled d can be determined by the formula $d = 3.5(s - 5)$. Solve for s.

Equation	Explanation	Reason
$d = 3.5(s - 5)$	Write original equation.	**Given**
$d =$ $3.5s - 17.5$	Multiply.	Distributive Property
$d +$ 17.5 $=$ $3.5s$	Add 17.5 to each side.	Addition **Property of Equality**
$\frac{d + 17.5}{3.5} = s$	Divide each side by 3.5.	Division **Property of Equality**

REFLEXIVE PROPERTY OF EQUALITY

Real Numbers For any real number a, $a = a$.

Segment Length For any segment AB, $AB = AB$.

Angle Measure For any angle A, $m\angle A = m\angle A$.

SYMMETRIC PROPERTY OF EQUALITY

Real Numbers For any real numbers a and b, if $a = b$, then $b = a$.

Segment Length For any segments AB and CD, if $AB = CD$, then $CD = AB$.

Angle Measure For any angles A and B, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

TRANSITIVE PROPERTY OF EQUALITY

Real Numbers For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Segment Length For any segments AB, CD, and EF, if $AB = CD$ and $CD = EF$, then $AB = EF$.

Angle Measure For any angles A, B, and C, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Your Notes

Example 4 ***Use properties of equality***

Show that $CF = AD$.

Equation	Reason
$AB =$ EF	Given
$BC =$ DE	Given
$AC = AB + BC$	Segment Addition Postulate
$DF =$ DE $+$ EF	Segment Addition Postulate
$DF = BC + AB$	Substitution Property of Equality
$DF =$ AC	Transitive Property of Equality
$DF + CD =$ AC $+ CD$	Addition Property of Equality
CF $=$ AD	Substitution Property of Equality

✔ ***Checkpoint*** **Complete the following exercises. In Exercises 4–6, name the property of equality that the statement illustrates.**

3. Suppose the equation in Example 3 is $d = 5(s + 3)$. Solve for s. Write a reason for each step.

$d = 5(s + 3)$	Given
$d = 5s + 15$	Distributive Property
$d - 15 = 5s$	Subtraction Property of Equality
$\frac{d - 15}{5} = s$	Division Property of Equality

4. If $GH = JK$, then $JK = GH$.

Symmetric Property of Equality for Segment Length

5. If $r = s$, and $s = 44$, then $r = 44$.

Transitive Property of Equality for Real Numbers

6. $m\angle N = m\angle N$

Reflexive Property of Equality for Angle Measure

Homework

2.6 Prove Statements about Segments and Angles

Goal • Write proofs using geometric theorems.

Your Notes

VOCABULARY

Proof A proof is a logical argument that shows a statement is true.

Two-column proof A two-column proof has numbered statements and corresponding reasons that show an argument in logical order.

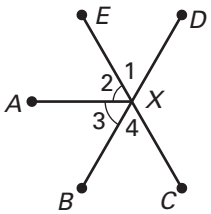
Theorem A theorem is a statement that can be proven.

Example 1 ***Write a two-column proof***

Use the diagram to prove $m\angle 1 = m\angle 4$.

Given $m\angle 2 = m\angle 3$, $m\angle AXD = m\angle AXC$

Prove $m\angle 1 = m\angle 4$



Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Statements	Reasons
1. $m\angle AXC = m\angle AXD$	1. Given
2. $m\angle AXD = m\angle$ 1 $+ m\angle$ 2	2. Angle Addition Postulate
3. $m\angle AXC = m\angle$ 3 $+ m\angle$ 4	3. Angle Addition Postulate
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4. Substitution Property of Equality
5. $m\angle 2 = m\angle 3$	5. Given
6. $m\angle 1 + m\angle$ 3 $= m\angle 3 + m\angle 4$	6. Substitution Property of Equality
7. $m\angle 1 = m\angle 4$	7. Subtraction Property of Equality

Your Notes

THEOREM 2.1 CONGRUENCE OF SEGMENTS

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

THEOREM 2.2 CONGRUENCE OF ANGLES

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Example 2 ***Name the property shown***

Name the property illustrated by the statement.

If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$.

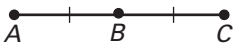
Symmetric Property of Angle Congruence

Checkpoint **Complete the following exercises.**

1. Three steps of a proof are shown. Give the reasons for the last two steps.

Given $BC = AB$

Prove $AC = AB + AB$



Statements	Reasons
1. $BC = AB$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = AB + AB$	3. Substitution Property of Equality

2. Name the property illustrated by the statement.
If $\angle H \cong \angle T$ and $\angle T \cong \angle B$, then $\angle H \cong \angle B$.

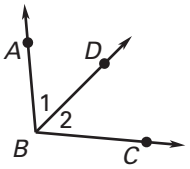
Transitive Property of Angle Congruence

Your Notes

Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

Example 3 ***Use properties of equality***

If you know that $\overrightarrow{BD}$ bisects $\angle ABC$, prove that $m\angle ABC$ is two times $m\angle 1$.



Given $\overrightarrow{BD}$ bisects $\angle ABC$.

Prove $m\angle ABC = 2 \cdot m\angle 1$

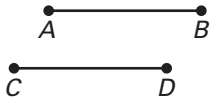
Statements	Reasons
1. $\overrightarrow{BD}$ bisects $\angle ABC$.	1. Given
2. $\angle 1 \cong \angle 2$	2. Definition of angle bisector
3. $m\angle 1 = m\angle 2$	3. Definition of congruent angles
4. $m\angle 1 + m\angle 2 = m\angle ABC$	4. Angle Addition Postulate
5. $m\angle 1 + m\angle$ 1 $= m\angle ABC$	5. Substitution Property of Equality
6. $2 \cdot m\angle 1 = m\angle ABC$	6. Distributive Property

CONCEPT SUMMARY: WRITING A TWO-COLUMN PROOF

Proof of the Symmetric Property of Segment Congruence

Given $\overline{AB} \cong \overline{CD}$

Prove $\overline{CD} \cong \overline{AB}$



Copy or draw diagrams and label information to help develop proofs.

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $AB = CD$	2. Definition of congruent segments
3. $CD = AB$	3. Symmetric Property of Equality
4. $\overline{CD} \cong \overline{AB}$	4. Definition of congruent segments

Statements based on facts that you know or conclusions from deductive reasoning

The number of statements will vary.

Remember to give a reason for the last statement.

Definitions, postulates, or proven theorems that allow you to state the corresponding statement.

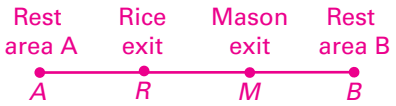
Your Notes

Example 4 ***Solve a multi-step problem***

Interstate There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and the Mason exit. The distance between rest area B and the Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

Solution

Step 1 **Draw** a diagram.



Step 2 **Draw** diagrams showing relationships.

Step 3 **Write** a proof.

Given R is the midpoint of $\overline{AM}$, $MB = AR$.

Prove M is the midpoint of $\overline{RB}$.

Statements	Reasons
1. R is the midpoint of $\overline{AM}$, $MB = AR$.	1. Given
2. $\overline{AR} \cong \overline{RM}$	2. Definition of midpoint
3. $AR = RM$	3. Definition of congruent segments
4. $MB = RM$	4. Transitive Property of Congruence
5. $\overline{MB} \cong \overline{RM}$	5. Definition of congruent segments
6. M is the midpoint of $\overline{RB}$.	6. Definition of midpoint

Checkpoint **Complete the following exercise.**

Homework

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

Rest area A and rest area B

2.7 Prove Angle Pair Relationships

Goal • Use properties of special pairs of angles.

Your Notes

THEOREM 2.3 RIGHT ANGLES CONGRUENCE THEOREM

All right angles are congruent.

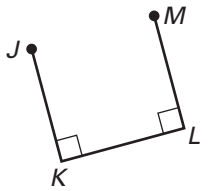
Example 1 ***Use right angle congruence***

The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show that the angles are right angles.

Write a proof.

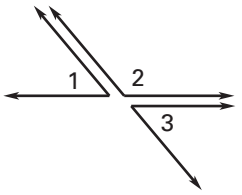
Given $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$

Prove $\angle K \cong \angle L$



Statements	Reasons
1. $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$	1. Given
2. $\angle K$ and $\angle L$ are right angles.	2. Definition of perpendicular lines
3. $\angle K \cong \angle L$	3. Right Angles Congruence Theorem

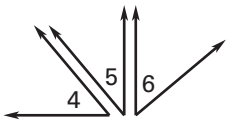
THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM



If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM



If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

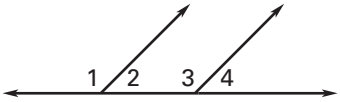
If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Your Notes

Example 2 ***Use the Congruent Supplements Theorem***

Write a proof.

Given $\angle 1$ and $\angle 2$ are supplements.
$\angle 1$ and $\angle 4$ are supplements.
$m\angle 2 = 45°$



Prove $m\angle 4 = 45°$

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplements. $\angle 1$ and $\angle 4$ are supplements.	1. Given
2. $\angle 2 \cong \angle 4$	2. Congruent Supplements Theorem
3. $m\angle 2 = m\angle 4$	3. Definition of congruent angles
4. $m\angle 2 = 45°$	4. Given
5. $m\angle 4 = 45°$	5. Substitution Property of Equality

✔ ***Checkpoint*** **Complete the following exercises.**

1. In Example 1, suppose you are given that $\angle K \cong \angle L$. Can you use the Right Angles Congruence Theorem to prove that $\angle K$ and $\angle L$ are right angles? *Explain*.

No, you cannot prove that $\angle K$ and $\angle L$ are right angles, because the converse of the Right Angles Congruence Theorem is not always true.

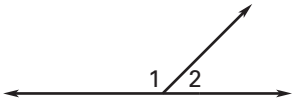
2. Suppose $\angle A$ and $\angle B$ are complements, and $\angle A$ and $\angle C$ are complements. Can $\angle B$ and $\angle C$ be supplements? *Explain*.

No, $\angle B$ and $\angle C$ are complements by the Congruent Complements Theorem, so they cannot be supplements.

Your Notes

POSTULATE 12 LINEAR PAIR POSTULATE

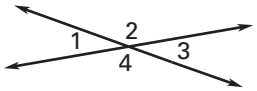
If two angles form a linear pair, then they are supplementary.



$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 =$ 180°.

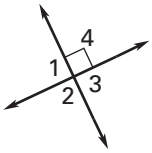
THEOREM 2.6 VERTICAL ANGLES CONGRUENCE THEOREM

Vertical angles are congruent.



Example 3 ***Determine whether a proof is valid.***

Tell if the proof is valid, or explain how to make it valid.



Given $\angle 4$ is a right angle.

Prove $\angle 2$ and $\angle 4$ are supplementary.

Statements	Reasons
1. $\angle 4$ is a right angle.	1. Given
2. $m\angle 4 = 90°$	2. Definition of a right angle
3. $m\angle 2 = 90°$	3. Vertical Angles Congruence Theorem
4. $\angle 2$ and $\angle 4$ are supplementary.	4. $m\angle 2 + m\angle 4 = 180°$

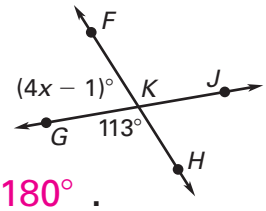
The proof is not valid. The Vertical Angle Congruence Theorem proves that $\angle 2$ $\cong$ $\angle 4$. It does not let you immediately conclude that $m\angle 2 = 90°$**. Here is a valid proof.**

Statements	Reasons
1. $\angle 4$ is a right angle.	1. Given
2. $m\angle 4 = 90°$	2. Definition of a right angle
3. $\angle 2 \cong \angle 4$	3. Vertical Angles Congruence Theorem
4. $m\angle 2 = m\angle 4$	4. Definition of $\cong$ angles
5. $m\angle 2 = 90°$	5. Substitution Property of Equality
6. $\angle 2$ and $\angle 4$ are supplementary.	6. $m\angle 2 + m\angle 4 = 180°$

Your Notes

Example 4 ***Find angle measures***

Find $m\angle FKG$.



Solution

Because $m\angle FKG$ and $m\angle GKH$ form a linear pair, the sum of their measures is 180°.

$(4x - 1)° + 113° =$ 180°	**Write equation.**
$4x +$ 112 $=$ 180	**Simplify.**
$4x =$ 68	**Subtract.**
$x =$ 17	**Divide.**

Use $x =$ 17 to find $m\angle FKG$.

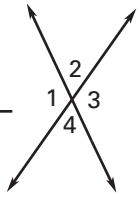
$m\angle FKG = (4x - 1)°$	**Write equation.**
$= [4($ 17 $) - 1]°$	**Substitute** 17 **for** x.
$= [$ 68 $- 1]°$	**Multiply.**
$=$ 67°	**Simplify.**

The measure of $\angle FKG$ is 67°.

Checkpoint **Complete the following exercises.**

3. If $m\angle 4 = 63°$, find $m\angle 1$ and $m\angle 2$.

$m\angle 1 = 117°$, $m\angle 2 = 63°$

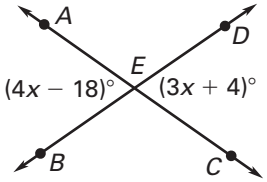


4. Explain why the definition of congruent angles is necessary for the proof in Example 3 to be logically valid.

Sample answer: The Vertical Angles Congruence Theorem only proves that the angles are congruent. The definition of congruent angles is necessary to prove that their measures are equal.

5. Find $m\angle AEB$.

$m\angle AEB = 70°$



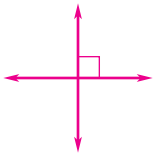
Homework

Words to Review

Give an example of the vocabulary word.

Conjecture A conjecture is an unproven statement that is based on observations.	Inductive reasoning You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.
Counterexample A counterexample is a specific case for which the conjecture is false.	Conditional statement Leaves change color in fall.
If-then form If leaves are changing color, then it is fall.	Hypothesis If leaves are changing color . . .
Conclusion . . . then it is fall.	Negation The leaves are not changing color.
Converse If it is fall, then leaves are changing color.	Inverse If leaves are not changing color, then it is not fall.
Contrapositive If it is not fall, then leaves are not changing color.	Equivalent statements A conditional statement is equivalent to its contrapositive. The inverse and converse of a conditional statement are also equivalent.

Perpendicular lines



Biconditional statement

The value of x is 5 if and only if $x - 3 = 2$.

Deductive reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

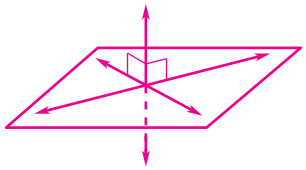
Logically equivalent

If p, then q.
If not q, then not p.

Truth table/Truth values

p	q	$q \rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

Line perpendicular to a plane



Proof

A logical argument that shows a statement is true.

Theorem

Vertical angles are congruent.

Two-column proof

Given $\angle 1 \cong \angle 2$, $m\angle 1 = 60°$
Prove $m\angle 2 = 60°$

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent $\angle$s
3. $m\angle 1 = 60°$	3. Given
4. $m\angle 2 = 60°$	4. Transitive Property of =

Review your notes and Chapter 2 by using the Chapter Review on pages 137–140 of your textbook.

3.1 Identify Pairs of Lines and Angles

Goal • Identify angle pairs formed by three intersecting lines.

Your Notes

VOCABULARY

Parallel lines Two lines are parallel lines if they do not intersect and are coplanar.

Skew lines Two lines are skew lines if they do not intersect and are not coplanar.

Parallel planes Two planes that do not intersect are parallel planes.

Transversal A transversal is a line that intersects two or more coplanar lines at different points.

Corresponding angles Two angles are corresponding angles if they have corresponding positions.

Alternate interior angles Two angles are alternate interior angles if they lie between the two lines and on opposite sides of the transversal.

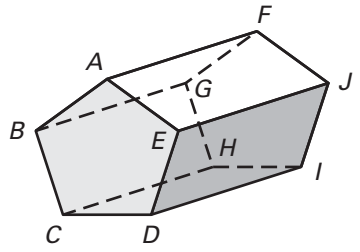
Alternate exterior angles Two angles are alternate exterior angles if they lie outside the two lines and on opposite sides of the transversal.

Consecutive interior angles Two angles are consecutive interior angles if they lie between the two lines and on the same side of the transversal.

Your Notes

Example 1 ***Identify relationships in space***

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?



a. Line(s) parallel to $\overleftrightarrow{AF}$ and containing point *E*

b. Line(s) skew to $\overleftrightarrow{AF}$ and containing point *E*

c. Line(s) perpendicular to $\overleftrightarrow{AF}$ and containing point *E*

d. Plane(s) parallel to plane *FGH* and containing point *E*

Solution

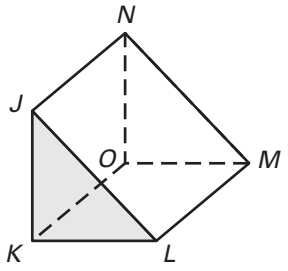
a. $\overleftrightarrow{EJ}$, $\overleftrightarrow{BG}$, $\overleftrightarrow{CH}$, and $\overleftrightarrow{DI}$ all appear parallel to $\overleftrightarrow{AF}$, but only $\overleftrightarrow{EJ}$ contains point *E*.

b. $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{GH}$, $\overleftrightarrow{HI}$, and $\overleftrightarrow{IJ}$ all appear skew to $\overleftrightarrow{AF}$, but only $\overleftrightarrow{DE}$ contains point *E*.

c. $\overleftrightarrow{AB}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{FG}$, and $\overleftrightarrow{FJ}$ all appear perpendicular to $\overleftrightarrow{AF}$, but only $\overleftrightarrow{AE}$ contains point *E*.

d. Plane *ABC* appears parallel to plane *FGH* and contains point *E*.

✔ ***Checkpoint*** **Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?**



1. parallel to $\overleftrightarrow{MN}$ and contains *J*

$\overleftrightarrow{JL}$

2. skew to $\overleftrightarrow{MN}$ and contains *J*

$\overleftrightarrow{KJ}$

3. perpendicular to $\overleftrightarrow{MN}$ and contains *J*

$\overleftrightarrow{JN}$

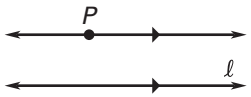
4. Name the plane that contains *J* and appears to be parallel to plane *MNO*.

plane *JKL*

Your Notes

POSTULATE 13 PARALLEL POSTULATE

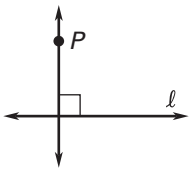
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

POSTULATE 14 PERPENDICULAR POSTULATE

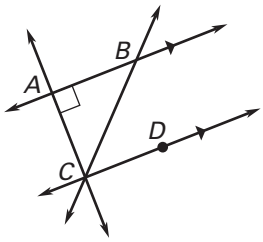
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

Example 2 ***Identify parallel and perpendicular lines***

Use the diagram at the right to answer each question.



a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$? *Explain*.

Solution

a. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

b. $\overleftrightarrow{AB} \perp \overleftrightarrow{AC}$

c. $\overleftrightarrow{AB}$ is not perpendicular to $\overleftrightarrow{BC}$, because $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AC}$ and by the Perpendicular Postulate there is exactly one line perpendicular to $\overleftrightarrow{AB}$ through C.

Checkpoint **Complete the following exercise.**

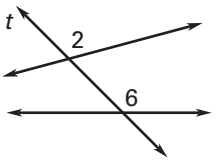
5. In Example 2, can you use the Perpendicular Postulate to show that $\overleftrightarrow{AC} \perp \overleftrightarrow{CD}$? *Explain*.

No, there is no right angle symbol at C so you do not know if $\overline{AC} \perp \overline{CD}$.

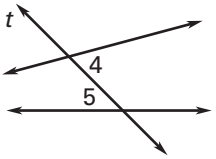
Your Notes

ANGLES FORMED BY TRANSVERSALS

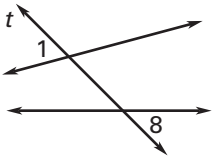
Two angles are corresponding angles if they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are alternate interior angles if they lie between the two lines and on opposite sides of the transversal.

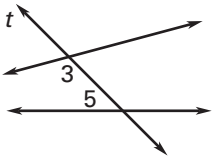


Two angles are alternate exterior angles if they lie outside the two lines and on opposite sides of the transversal.



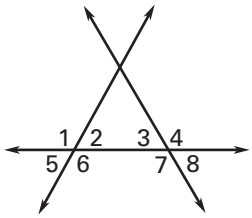
Another name for consecutive interior angles is same-side interior angles.

Two angles are consecutive interior angles if they lie between the two lines and on the same side of the transversal.



Example 3 ***Identify angle relationships***

Identify all pairs of (a) corresponding angles, (b) alternate interior angles, (c) alternate exterior angles, and (d) consecutive interior angles.



a. ∠1 and ∠3, ∠2 and ∠4, ∠5 and ∠7, ∠6 and ∠8

b. ∠2 and ∠7, ∠6 and ∠3

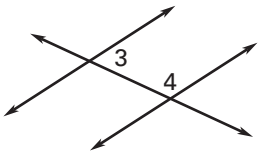
c. ∠5 and ∠4, ∠1 and ∠8

d. ∠2 and ∠3, ∠6 and ∠7

Homework

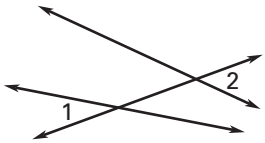
✔ ***Checkpoint*** **Classify the pair of numbered angles.**

6.



Consecutive interior angles

7.



Alternate exterior angles

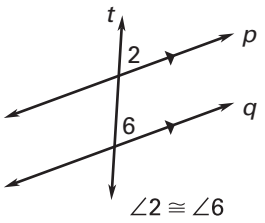
3.2 Use Parallel Lines and Transversals

Goal • Use angles formed by parallel lines and transversals.

Your Notes

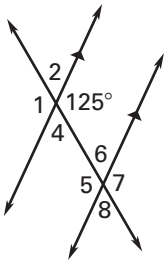
POSTULATE 15 CORRESPONDING ANGLES POSTULATE

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



Example 1 ***Identify congruent angles***

The measure of three of the numbered angles is 125°. Identify the angles. *Explain* your reasoning.



Solution

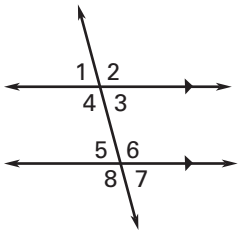
By the Corresponding Angles Postulate, $m\angle 7$ = 125°.

Using the Vertical Angles Congruence Theorem, $m\angle 1$ = 125°.

Because ∠1 and ∠5 are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 5$ = 125°.

✔ ***Checkpoint*** **Complete the following exercise using the diagram shown.**

1. If $m\angle 7$ = 75°, find $m\angle 1$, $m\angle 3$, and $m\angle 5$. Tell which postulate or theorem you use in each case.

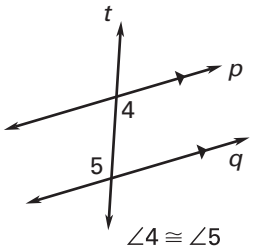


$m\angle 3$ = 75°, Corresponding Angles Postulate; $m\angle 5$ = 75°, Vertical Angles Congruence Theorem; $m\angle 1$ = 75°, Corresponding Angles Postulate

Your Notes

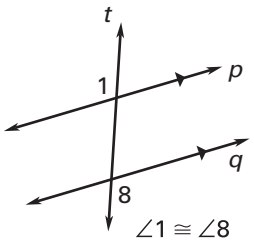
THEOREM 3.1 ALTERNATE INTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



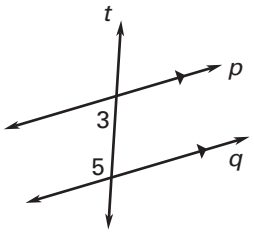
THEOREM 3.2 ALTERNATE EXTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



THEOREM 3.3 CONSECUTIVE INTERIOR ANGLES THEOREM

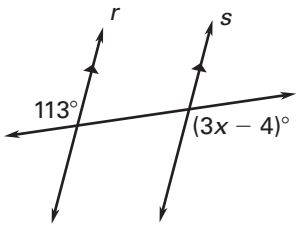
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



∠3 and ∠5 are supplementary.

Example 2 ***Use properties of parallel lines***

Find the value of *x*.



Solution

Lines *r* and *s* are parallel, so you can use the theorems about parallel lines.

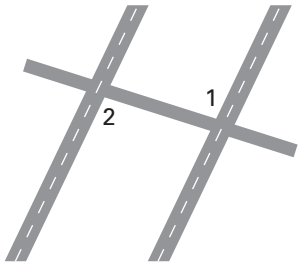
$113° = (3x - 4)°$	Alternate Exterior Angles Theorem
$117 = 3x$	**Add 4 to each side.**
$39 = x$	**Divide each side by 3.**

The value of *x* is 39.

Your Notes

Example 3 ***Solve a real-world problem***

Runways A taxiway is being constructed that intersects two parallel runways at an airport. You know that $m\angle 2 = 98°$. What is $m\angle 1$? How do you know?

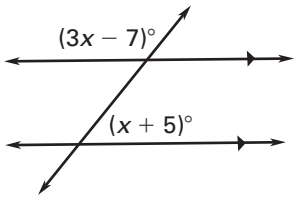


Solution

Because the runways are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 98°$.

✔ ***Checkpoint*** **Complete the following exercises.**

2. Find the value of x.



$x = 45.5$

3. In Example 3, suppose $\angle 3$ is the consecutive interior angle with $\angle 2$. What is $m\angle 3$?

82°

Homework

3.3 Prove Lines are Parallel

Goal • Use angle relationships to prove that lines are parallel.

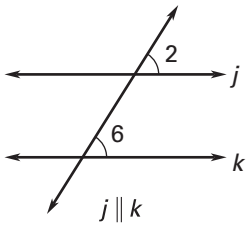
Your Notes

VOCABULARY

Paragraph proof A proof can be written in paragraph form, called a paragraph proof.

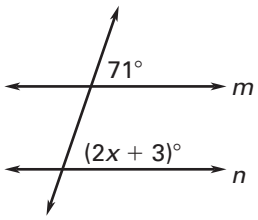
POSTULATE 16 CORRESPONDING ANGLES CONVERSE

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



Example 1 ***Apply the Corresponding Angles Converse***

Find the value of x that makes $m \parallel n$.



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

$(2x + 3)° = 71°$ **Use Postulate 16 to write an equation.**

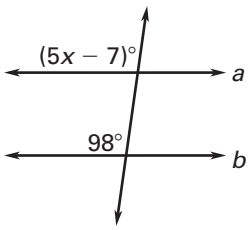
$2x = 68$ **Subtract 3 from each side.**

$x = 34$ **Divide each side by 2.**

The lines m and n are parallel when $x = 34$.

✔ ***Checkpoint*** **Find the value of x that makes $a \parallel b$.**

1.

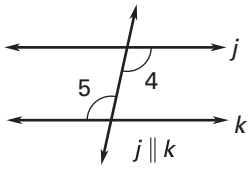


$x = 21$

Your Notes

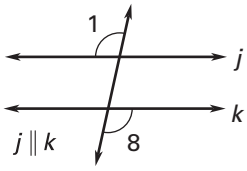
THEOREM 3.4 ALTERNATE INTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.



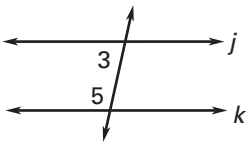
THEOREM 3.5 ALTERNATE EXTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.



THEOREM 3.6 CONSECUTIVE INTERIOR ANGLES CONVERSE

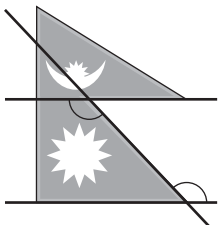
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



If ∠3 and ∠5 are supplementary, then $j \parallel k$.

Example 2 ***Solve a real-world problem***

Flags How can you tell whether the sides of the flag of Nepal are parallel?

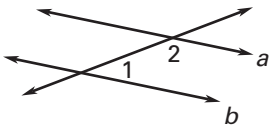


Solution

Because the alternate interior angles are congruent, you know that the sides of the flag are parallel.

✔ ***Checkpoint*** **Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.**

2. $m\angle 1 + m\angle 2 = 180°$

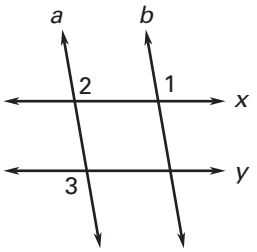


Yes, you can use the Consecutive Interior Angles Converse to prove $a \parallel b$.

Your Notes

Example 3 ***Write a paragraph proof***

In the figure, $a \parallel b$ and $\angle 1$ is congruent to $\angle 3$. Prove $x \parallel y$.

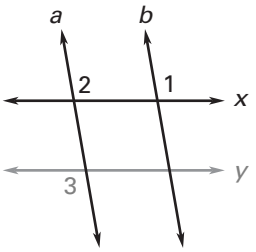


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

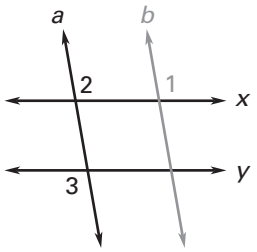
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $a \parallel b$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$ then $x \parallel y$.

In paragraph proofs, transitional words such as *so, then*, and *therefore* help to make the logic clear.

Plan in Action

a. It is given that $a \parallel b$, so by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. Therefore, by the Alternate Exterior Angles Converse, $x \parallel y$.

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose it is given that $\angle 1 \cong \angle 3$ and $x \parallel y$. Complete the following paragraph proof showing that $a \parallel b$.

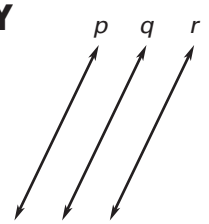
It is given that $x \parallel y$. By the Exterior Angles Postulate, $\angle 2 \cong \angle 3$.

It is also given that $\angle 1 \cong \angle 3$. Then $\angle 1 \cong \angle 2$ by the Transitive Property of Congruence for angles. Therefore, by the Corresponding Angles Converse, $a \parallel b$.

Your Notes

THEOREM 3.7 TRANSITIVE PROPERTY OF PARALLEL LINES

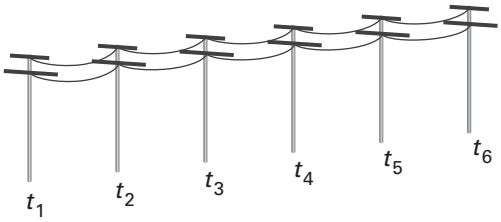
If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Example 4 ***Use the Transitive Property of Parallel Lines***

Utility poles Each utility pole shown is parallel to the pole immediately to its right. *Explain* why the leftmost pole is parallel to the rightmost pole.



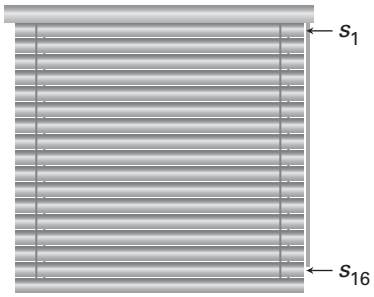
When you name several similar items, you can use one variable with subscripts to keep track of the items.

Solution

The poles from left to right can be named $t_1, t_2, t_3, \ldots, t_6$. Each pole is parallel to the one to its right, so $t_1 \parallel$ t_2, $t_2 \parallel$ t_3, and so on. Then $t_1 \parallel t_3$ by the Transitive Property of Parallel Lines. Similarly, because $t_3 \parallel t_4$, it follows that $t_1 \parallel$ t_4. By continuing this reasoning, $t_1 \parallel$ t_6. So, the leftmost pole is parallel to the rightmost pole.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Each horizontal piece of the window blinds shown is called a slat. Each slat is parallel to the slat immediately below it. *Explain* why the top slat is parallel to the bottom slat.



The slats from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{16}$. Each slat is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{16}$. So, the top slat is parallel to the bottom slat.

Homework

3.4 Find and Use Slopes of Lines

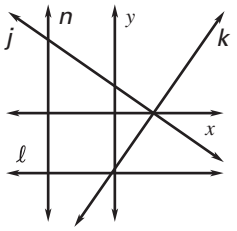
Goal • Find and compare slopes of lines.

Your Notes

VOCABULARY

Slope The slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line.

SLOPE OF LINES IN THE COORDINATE PLANE



Negative slope: falls from left to right, as in line *j*

Positive slope: rises from left to right, as in line *k*

Undefined slope: vertical, as in line *n*

Zero slope (slope of 0): horizontal, as in line ℓ

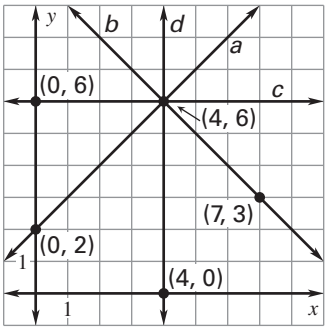
Slope

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Example 1 ***Find slopes of lines in a coordinate plane***

Find the slope of line *a* and line *c*.



Slope of line *a*:

$m = \frac{6 - \boxed{2}}{4 - \boxed{0}} = \frac{\boxed{4}}{\boxed{4}} = 1$

Slope of line *c*:

$m = \frac{6 - \boxed{6}}{4 - \boxed{0}} = \frac{\boxed{0}}{\boxed{4}} = 0$

✔ ***Checkpoint*** **Use the graph in Example 1. Find the slope of the line.**

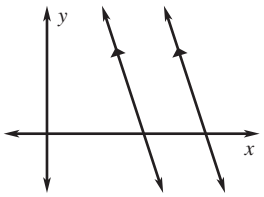
1. line *b* −1	**2.** line *d* undefined

Your Notes

POSTULATE 17 SLOPES OF PARALLEL LINES

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

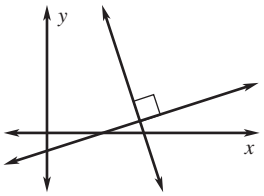


$m_1 = m_2$

POSTULATE 18 SLOPES OF PERPENDICULAR LINES

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

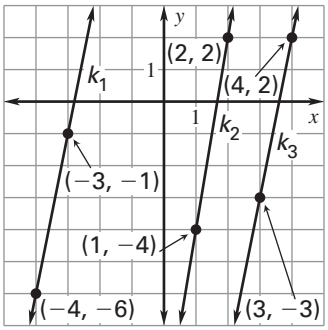


$m_1 \cdot m_2 = -1$

If the product of two numbers is -1, then the numbers are called *negative reciprocals*.

Example 2 ***Identify parallel lines***

Find the slope of each line. Which lines are parallel?



Solution

Find the slope of k_1.

$$m = \frac{-1-(-6)}{-3-(-4)} = \frac{5}{1} = 5$$

Find the slope of k_2.

$$m = \frac{2-(-4)}{2-1} = 6$$

Find the slope of k_3.

$$m = \frac{2-(-3)}{4-3} = 5$$

Compare the slopes. Because k_1 and k_3 have the same slope, they are parallel. The slope of k_2 is different, so k_2 is not parallel to the other lines.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Line c passes through $(2, -2)$ and $(5, 7)$. Line d passes through $(-3, 4)$ and $(1, -8)$. Are the two lines parallel? *Explain* how you know.

No; the slope of c is not equal to the slope of d.

Your Notes

Example 3 *Draw a perpendicular line*

Line *h* passes through (1, −2) and (5, 6). Graph the line perpendicular to *h* that passes through the point (2, 5).

Step 1 **Find** the slope m_1 of *h* through (1, −2) and (5, 6).

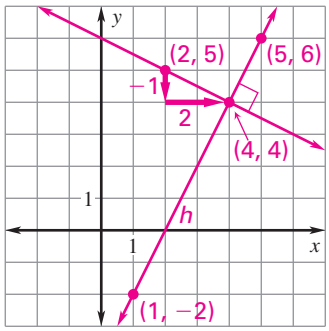
$$m_1 = \frac{6-(-2)}{5-1} = \frac{8}{4} = 2$$

Step 2 **Find** the slope m_2 of a line perpendicular to *h*.

$$2 \cdot m_2 = -1$$

$$m_2 = -\frac{1}{2}$$

Step 3 **Use** the rise and run to graph the line.



Given a point on a line and the line's slope, you can use the rise and run to find a second point and draw the line.

Example 4 *Use properties of a quadrilateral*

Quadrilateral *ABCD* has vertices *A*(1, 5), *B*(6, 7), C(8, 2), and *D*(3, 0). Is *ABCD* a *rectangle*, a *square*, or *neither*?

The distance *AB* between $A(x_1, y_1)$ and $B(x_2, y_2)$ can be expressed as $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Line	Find Slopes	Find Lengths
$\overline{AB}$	$\frac{7-5}{6-1} = \frac{2}{5}$	$\sqrt{(6-1)^2 + (7-5)^2} = \sqrt{29}$
$\overline{BC}$	$\frac{2-7}{8-6} = -\frac{5}{2}$	$\sqrt{(8-6)^2 + (2-7)^2} = \sqrt{29}$
$\overline{CD}$	$\frac{0-2}{3-8} = \frac{-2}{-5} = \frac{2}{5}$	$\sqrt{(3-8)^2 + (0-2)^2} = \sqrt{29}$
$\overline{DA}$	$\frac{5-0}{1-3} = -\frac{5}{2}$	$\sqrt{(1-3)^2 + (5-0)^2} = \sqrt{29}$

Multiplying slopes of consecutive sides yields −1, so *ABCD* has four right angles. The four sides are all the same length, so *ABCD* is a square.

Homework

✔ ***Checkpoint*** **Complete the following exercise**

4. Quadrilateral *ABCD* has vertices *A*(−1, 1), *B*(4, 3), *C*(6, −2), *D*(1, −4). Show it has congruent diagonals.

$AC = BD = \sqrt{58}$, so $\overline{AC} \cong \overline{BD}$.

3.5 Write and Graph Equations of Lines

Goal • Find equations of lines.

Your Notes

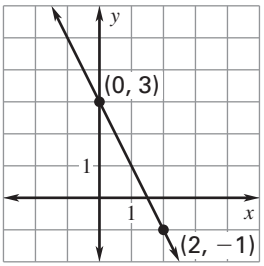
VOCABULARY

Slope-intercept form The general form of a linear equation in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Standard form The general form of a linear equation in standard form is $Ax + By = C$, where A and B are not both zero.

Example 1 ***Write an equation of a line from a graph***

Write an equation of the line in slope-intercept form.



Solution

Step 1 **Find** the slope. Choose two points on the graph of the line, (0, 3) and (2, −1).

$$m = \frac{3 - (-1)}{0 - 2} = \frac{4}{-2} = -2$$

Step 2 **Find** the y-intercept. The line intersects the y-axis at the point (0, 3), so the y-intercept is 3.

Step 3 **Write** the equation.

$y = mx + b$ **Use slope-intercept form.**

$y = -2x + 3$ **Substitute** −2 **for** m **and** 3 **for** b.

Your Notes

The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

Example 2 ***Write an equation of a parallel line***

Write an equation of the line passing through the point (1, −1) that is parallel to the line with the equation $y = 2x - 1$.

Solution

Step 1 **Find** the slope m. The slope of a line parallel to $y = 2x - 1$ is the same as the given line, so the slope is 2.

Step 2 **Find** the y-intercept b by using $m =$ 2 and $(x, y) =$ (1, −1).

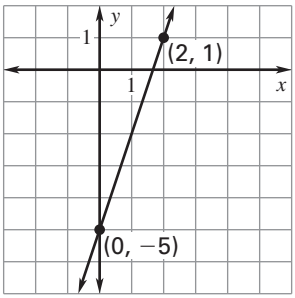
$y = mx + b$	**Use slope-intercept form.**
$-1 = 2(1) + b$	**Substitute for *x*, *y*, and *m*.**
$-3 = b$	**Solve for *b*.**

Because $m =$ 2 and $b =$ −3, an equation of the line is $y =$ $2x - 3$.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Write an equation of the line in the graph at the right.

$y = 3x - 5$



2. Write an equation of the line that passes through the point (−2, 5) and is parallel to the line with the equation $y = -2x + 3$.

$y = -2x + 1$

Your Notes

Example 3 ***Write an equation of a perpendicular line***

Write an equation of the line *j* passing through the point (3, 2) that is perpendicular to the line *k* with the equation $y = -3x + 1$.

Solution

Step 1 **Find** the slope m of line j. The slope of k is -3.

$-3 \cdot m = -1$ **The product of the slopes of perpendicular lines is -1.**

$m = \frac{1}{3}$ **Divide each side by -3.**

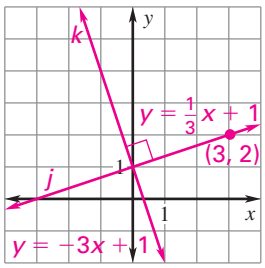
Step 2 **Find** the y-intercept b by using $m = \frac{1}{3}$ and $(x, y) = (3, 2)$.

$y = mx + b$ **Use slope-intercept form.**

$2 = \frac{1}{3}(3) + b$ **Substitute for x, y, and m.**

$1 = b$ **Solve for b.**

Because $m = \frac{1}{3}$ and $b = 1$, an equation of line j is $y = \frac{1}{3}x + 1$.



You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.

✔ ***Checkpoint*** **Complete the following exercise.**

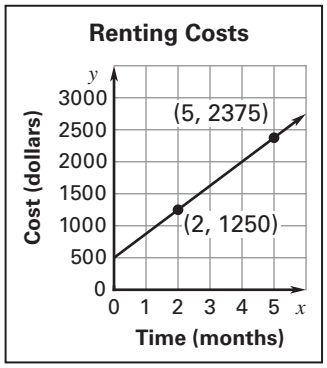
3. Write an equation of the line passing through the point $(-8, -2)$ that is perpendicular to the line with the equation $y = 4x - 3$.

$y = -\frac{1}{4}x - 4$

Your Notes

Example 4 ***Write an equation of a line from a graph***

Rent The graph models the total cost of renting an apartment. Write an equation of the line. *Explain* the meaning of the slope and the *y*-intercept of the line.



Step 1 Find the slope.

$$m = \frac{2375 - 1250}{5 - 2}$$

$$= \frac{1125}{3} = 375$$

Step 2 Find the *y*-intercept. Use a point on the graph.

$y = mx + b$ **Use slope-intercept form.**

$1250 = 375 \cdot 2 + b$ **Substitute.**

$500 = b$ **Simplify.**

Step 3 Write the equation. Because $m =$ 375 and $b =$ 500, an equation is $y =$ $375x + 500$.

The equation $y =$ $375x + 500$ models the cost. The slope is the monthly rent, and the *y*-intercept is the initial cost to rent the apartment.

Example 5 ***Graph a line with equation in standard form***

Graph $2x + 3y = 6$.

The equation is in standard form, so use the intercepts.

Step 1 Find the intercepts.

To find the *x*-intercept, let $y =$ 0.

$2x + 3y = 6$

$2x + 3(0) = 6$

$x = 3$

To find the *y*-intercept, let $x =$ 0.

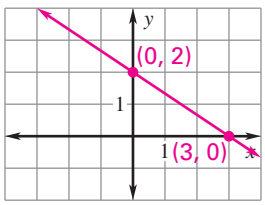
$2x + 3y = 6$

$2(0) + 3y = 6$

$y = 2$

Step 2 Graph the line.

The line intersects the axes at (3, 0) and (0, 2). Graph these points, then draw a line through the points.



Your Notes

Example 6 ***Solve a real-world problem***

Subscriptions You can buy a magazine at a store for $3. You can subscribe yearly to the magazine for a flat fee of $18. After how many magazines is the subscription a better buy?

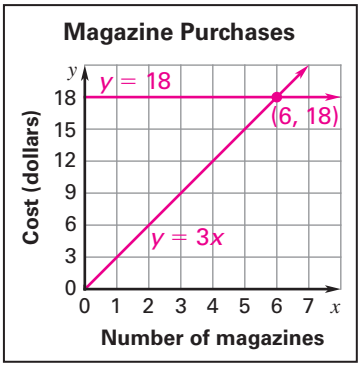
Solution

Step 1 **Model** each purchase with an equation.

Cost of yearly subscription: $y =$ 18

Cost of one magazine: $y =$ 3 x, where x represents the number of magazines

Step 2 **Graph** each equation.



The point of intersection is (6, 18). Using the graph, you can see that it is cheaper to buy magazines individually if you buy less than 6 magazines per year. If you buy more than 6 magazines per year, it is cheaper to buy a subscription.

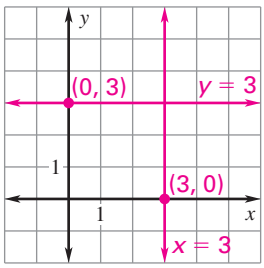
The point at which the costs are the same is sometimes called the *break-even point*.

✔ ***Checkpoint*** **Complete the following exercises.**

4. The equation $y = 650x + 425$ models the total cost of joining a health club for x years. What are the meaning of the slope and y-intercept of the line?

The slope is the cost per year, $650, and the y-intercept is the initiation fee, $425.

5. Graph $y = 3$ and $x = 3$.



6. In Example 6, suppose you can buy the magazine at a different store for $2.50. After how many magazines is the subscription the better buy?

after 7 magazines

Homework

3.6 Prove Theorems About Perpendicular Lines

Goal • Find the distance between a point and a line.

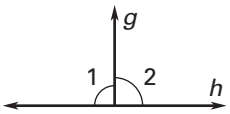
Your Notes

VOCABULARY

Distance from a point to a line The distance from a point to a line is the length of the perpendicular segment from the point to the line.

THEOREM 3.8

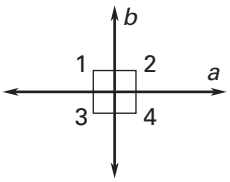
If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.



If $\angle 1 \cong \angle 2$, then $g \perp h$.

THEOREM 3.9

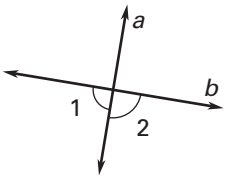
If two lines are perpendicular, then they intersect to form four right angles.



If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.

Example 1 ***Draw conclusions***

In the diagram at the right, $\angle 1 \cong \angle 2$. What can you conclude about *a* and *b*?

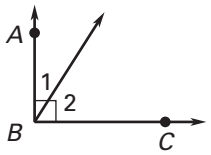


Solution

Lines *a* and *b* intersect to form a linear pair of congruent angles, $\angle 1$ and $\angle 2$. So, by Theorem 3.8, $a \perp b$.

Your Notes

THEOREM 3.10

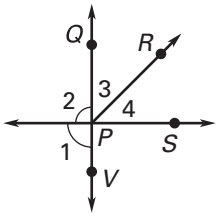


If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

Example 2 ***Write a proof***

In the diagram at the right, $\angle 1 \cong \angle 2$. Prove that $\angle 3$ and $\angle 4$ are complementary.



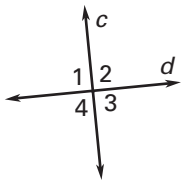
Given $\angle 1 \cong \angle 2$

Prove $\angle 3$ and $\angle 4$ are complementary.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overleftrightarrow{PS} \perp \overleftrightarrow{PQ}$	2. Theorem 3.8
3. $\angle 3$ and $\angle 4$ are complementary.	3. Theorem 3.10

Checkpoint **Complete the following exercises.**

1. If $c \perp d$, what do you know about the sum of the measures of $\angle 3$ and $\angle 4$? *Explain*.



Because $c \perp d$, angles 1, 2, 3, and 4 are right angles by Theorem 3.9. So, $m\angle 3 + m\angle 4 = 180°$.

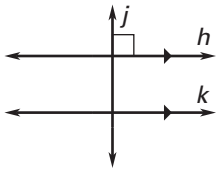
2. Using the diagram in Example 2, complete the following proof that $\angle QPS$ and $\angle 1$ are right angles.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overleftrightarrow{PS} \perp \overleftrightarrow{PQ}$	2. Theorem 3.8
3. $\angle QPS$ and $\angle 1$ are right angles.	3. Theorem 3.9

Your Notes

THEOREM 3.11 PERPENDICULAR TRANSVERSAL THEOREM

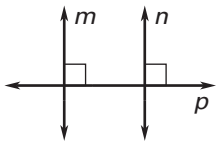
If a transversal is perpendicular to one of two parallel lines, then it is __perpendicular__ to the other.



If $h \parallel k$ and $j \perp h$, then j __$\perp$__ k.

THEOREM 3.12 LINES PERPENDICULAR TO A TRANSVERSAL THEOREM

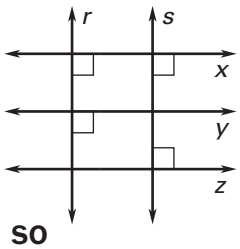
In a plane, if two lines are perpendicular to the same line, then they are __parallel__ to each other.



If $m \perp p$ and $n \perp p$, then m __$\parallel$__ n.

Example 3 ***Draw conclusions***

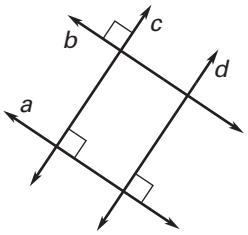
Determine which lines, if any, must be parallel in the diagram. *Explain* your reasoning.



Solution

Lines r and s are both perpendicular to __x__, so by Theorem 3.12, __$r \parallel s$__. Similarly, lines x and y are both perpendicular to r, so __$x \parallel y$__. Also, lines __x__ and __z__ are both perpendicular to s, so __$x \parallel z$__. Finally, because y and z are both parallel to __x__, you know that __$y \parallel z$__ by the Transitive Property of Parallel Lines.

✔ ***Checkpoint*** **Use the diagram to complete the following exercises.**



3. Is $c \parallel d$? *Explain*.

Yes, because c and d are both perpendicular to a, $c \parallel d$ by Theorem 3.12.

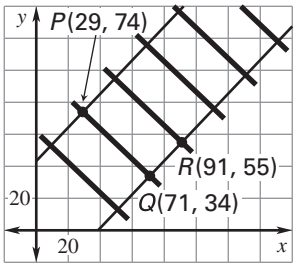
4. Is $b \perp d$? *Explain*.

Yes, because $b \perp c$, and $c \parallel d$ as explained in Exercise 3, then $b \perp d$ by Theorem 3.11.

Your Notes

Example 4 ***Find the distance between two parallel lines***

Railroads The section of broad gauge railroad track at the right is drawn on a graph where units are measured in inches. What is the distance from P to segment $\overline{QR}$?



Solution

You need to confirm that segment $\overline{RQ}$ is perpendicular to segment $\overline{PQ}$.

Using $Q(71, 34)$ and $R(91, 55)$, the slope of $\overline{QR}$ is

$$\frac{55 - \boxed{34}}{91 - \boxed{71}} = \frac{21}{20}$$

The segment $\overline{PQ}$ has a slope of

$$\frac{34 - \boxed{74}}{71 - \boxed{29}} = \frac{-40}{42} = -\frac{20}{21}$$

The segment $\overline{PQ}$ is perpendicular to segment $\overline{QR}$ so PQ is

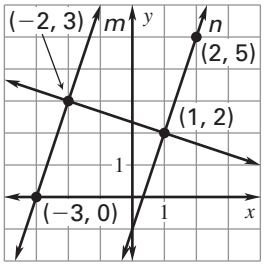
$d = \sqrt{(\underline{29 - 71})^2 + (\underline{74 - 34})^2} = \underline{58}$.

The distance from P to segment $\overline{QR}$ is <u>58 inches</u>.

✔ ***Checkpoint*** **Complete the following exercise.**

5. What is the approximate distance from line m to line n?

about 3.2 units



Homework

Focus On Reasoning

Use after Lesson 3.6

Taxicab Geometry

Goal • Find distances in non-Euclidean geometry.

Your Notes

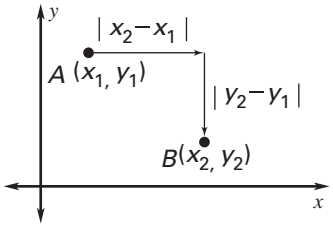
VOCABULARY

Taxicab geometry The non-Euclidean geometry that a taxicab or a pedestrian must obey.

TAXICAB DISTANCE

The distance between two points is the sum of the absolute values of the differences in their coordinates.

$AB = |x_2 - x_1| + |y_2 - y_1|$



Example 1 ***Find a taxicab distance***

Find the taxicab distance from *A*(4, 5) to *B*(−3, −4). Draw two different shortest paths from *A* to *B*.

Solution

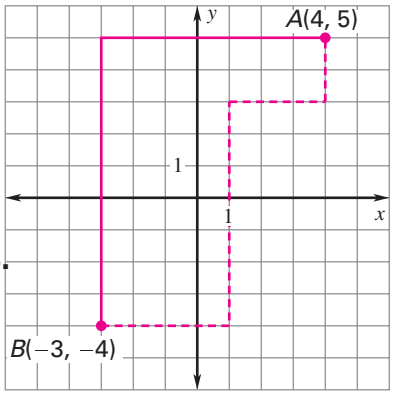
$AB = |x_2 - x_1| + |y_2 - y_1|$

$= |-3 - 4| + |-4 - 5|$

$= |-7| + |-9|$

$= 16$

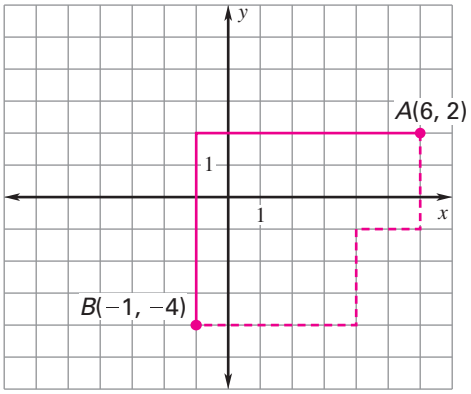
The shortest path is 16 blocks.



✔ ***Checkpoint*** **Find the taxicab distance from *A* to *B*. Draw two different shortest paths from *A* to *B***

1. $A(6, 2)$ to $B(-1, -4)$.

13

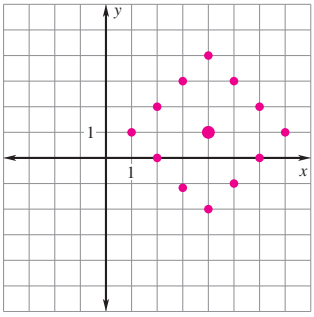


Example 2 ***Draw a taxicab circle.***

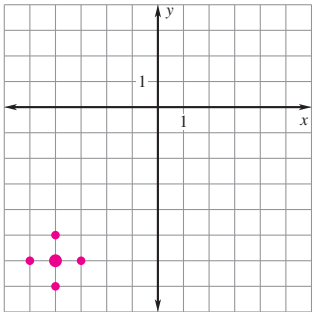
Draw the taxicab circle with the given radius *r* and center *C*.

Solution

a. $r = 3$, $C(4, 1)$

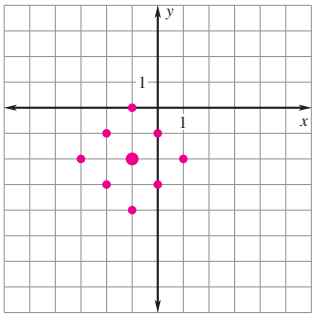


b. $r = 1$, $C(-4, -6)$

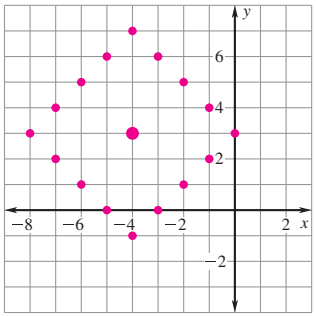


✔ ***Checkpoint*** **Draw the taxicab circle with the given radius *r* and center *C***

2. $r = 2$, $C(-1, -2)$



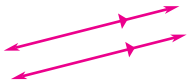
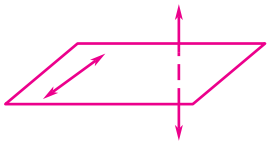
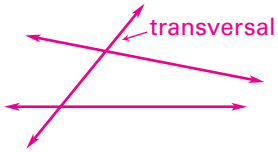
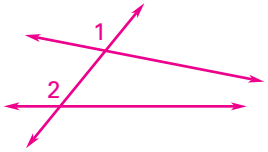
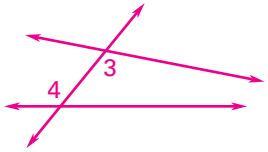
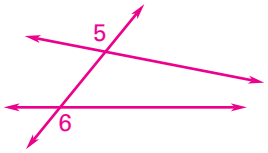
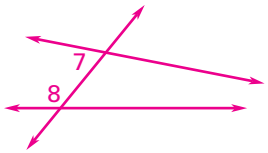
3. $r = 4$, $C(-4, 3)$

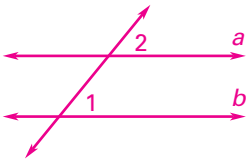
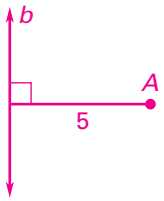
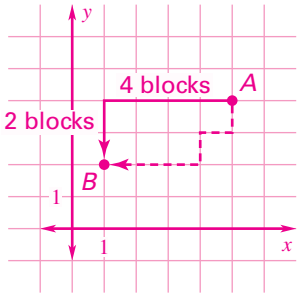


Homework

Words to Review

Give an example of the vocabulary word.

Parallel lines 	Skew lines 
Parallel planes 	Transversal 
Corresponding angles 	Alternate interior angles 
Alternate exterior angles 	Consecutive interior angles 

Paragraph proof  Given: $m\angle 1 = 50°$ $m\angle 2 = 50°$ Prove: $a \parallel b$ You are given that $m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$ by the definition of congruent angles. So, $a \parallel b$ by the Corresponding Angles Converse.	**Slope** The slope of $y = 8x - 7$ is 8.
Slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept.	**Standard form** $Ax + By = C$, where A and B are not both zero.
Distance from a point to a line.  The distance from point A to line b is 5.	**Taxicab geometry** 

Review your notes and Chapter 3 by using the Chapter Review on pages 209–212 of your textbook.

4.1 Apply Triangle Sum Properties

Goal • Classsify triangles and find measures of their angles.

Your Notes

VOCABULARY

Triangle A triangle is a polygon with three sides.

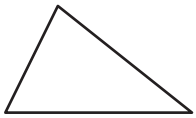
Interior angles When the sides of a polygon are extended, the original angles are the interior angles.

Exterior angles When the sides of a polygon are extended, the angles that form linear pairs with the interior angles are the exterior angles.

Corollary to a theorem A corollary to a theorem is a statement that can be proved easily using the theorem.

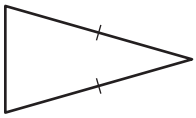
CLASSIFYING TRIANGLES BY SIDES

Scalene Triangle



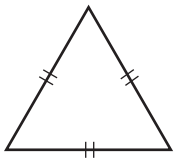
_No_ congruent sides

Isosceles Triangle



At least _2_ congruent sides

Equilateral Triangle

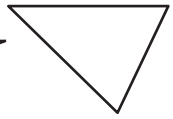


_3_ congruent sides

CLASSIFYING TRIANGLES BY ANGLES

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

Acute Triangle

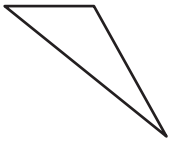


_3_ acute angles

Right Triangle

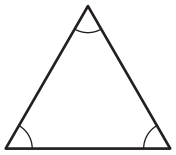
_1_ right angle

Obtuse Triangle



_1_ obtuse angle

Equiangular Triangle

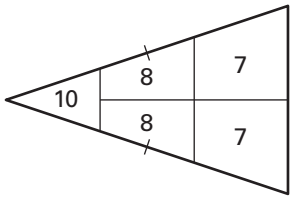


_3_ congruent angles

Your Notes

Example 1 ***Classify triangles by sides and by angles***

Shuffleboard Classify the triangular shape of the shuffleboard scoring area in the diagram by its sides and by measuring its angles.



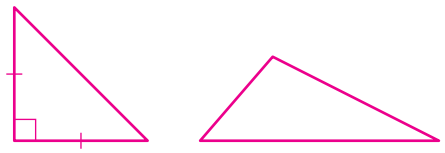
Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are about 72°, 72°, and 36°. It is an acute isosceles triangle.

✔ ***Checkpoint*** **Complete the folowing exercise.**

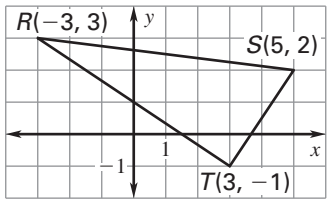
1. Draw an isosceles right triangle and an obtuse scalene triangle.

Sample Drawings:



Example 2 ***Classify a triangle in a coordinate plane***

Classify $\triangle RST$ by its sides. Then determine if the triangle is a right triangle.



Solution

Step 1 **Use** the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the side lengths.

$RT = \sqrt{(3 - (-3))^2 + (-1 - 3)^2} = \sqrt{52}$

$RS = \sqrt{(5 - (-3))^2 + (2 - 3)^2} = \sqrt{65}$

$ST = \sqrt{(3 - 5)^2 + (-1 - 2)^2} = \sqrt{13}$

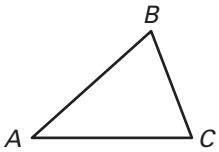
Step 2 **Check** for right angles. The slope of $\overline{RT}$ is $\frac{-1 - 3}{3 - (-3)} = -\frac{2}{3}$. The slope of $\overline{ST}$ is $\frac{-1 - 2}{3 - 5} = \frac{3}{2}$. The product of the slopes is -1, so $\overline{RT} \perp \overline{ST}$ and $\angle RTS$ is a right angle.

Therefore, $\triangle RST$ is a right scalene triangle.

Your Notes

THEOREM 4.1: TRIANGLE SUM THEOREM

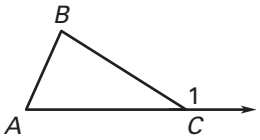
The sum of the measures of the interior angles of a triangle is 180°.



$m\angle A + m\angle B + m\angle C =$ 180°

THEOREM 4.2: EXTERIOR ANGLE THEOREM

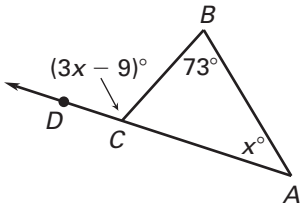
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.



$m\angle 1 = m\angle A + m\angle B$

Example 3 ***Find angle measure***

Use the diagram at the right to find the measure of $\angle DCB$.



Solution

Step 1 **Write** and solve an equation to find the value of x.

$(3x - 9)° = 73° + x°$ **Exterior Angle Theorem**

$x = 41°$ **Solve for x.**

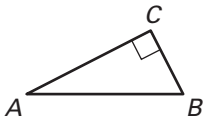
Step 2 **Substitute** 41 for x in $3x - 9$ to find $m\angle DCB$.

$3x - 9 = 3 \cdot 41 - 9 = 114$

The measure of $\angle DCB$ is 114°.

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

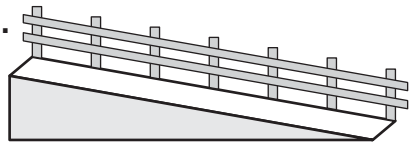


$m\angle A + m\angle B =$ 90°

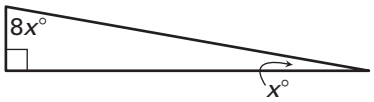
Your Notes

Example 4 ***Find angle measures from a verbal description***

Ramps The front face of the wheelchair ramp shown forms a right triangle. The measure of one acute angle in the triangle is eight times the measure of the other. Find the measure of each acute angle.



Solution



First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $8x°$.

Use the Corollary to the Triangle Sum Theorem to set up and solve an equation.

$x° + 8x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 10$ **Solve for *x*.**

So, the measures of the acute angles are $10°$ and $80°$.

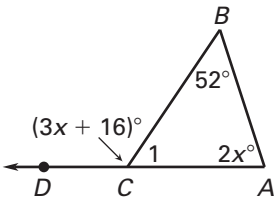
✔ ***Checkpoint*** **Complete the following exercises.**

2. Triangle *JKL* has vertices $J(-2, -1)$, $K(1, 3)$, and $L(5, 0)$. Classify it by its sides. Then determine if it is a right triangle.

isosceles triangle; right triangle

3. Find the measure of $\angle 1$ in the diagram shown.

56°



Homework

4. In Example 4, what is the measure of the obtuse angle formed between the ramp and a segment extending from the horizontal leg?

170°

4.2 Apply Congruence and Triangles

Goal • Identify congruent figures.

Your Notes

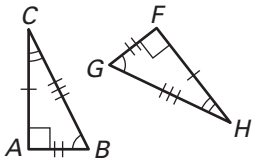
VOCABULARY

Congruent figures In two congruent figures, all the parts of one figure are congruent to the corresponding parts of the other figure.

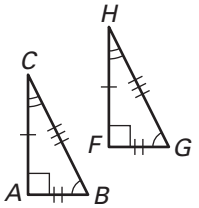
Corresponding parts In congruent polygons, the corresponding parts are the corresponding sides and the corresponding angles.

Example 1 ***Identify congruent parts***

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



To help you identify corresponding parts, turn $\triangle FGH$.



Solution

The diagram indicates that $\triangle ABC \cong \triangle$ FGH .

Corresponding angles $\angle A \cong$ $\angle F$, $\angle B \cong$ $\angle G$, $\angle C \cong$ $\angle H$

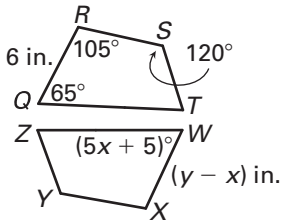
Corresponding sides $\overline{AB} \cong$ $\overline{FG}$, $\overline{BC} \cong$ $\overline{GH}$, $\overline{CA} \cong$ $\overline{HF}$

Example 2 ***Use properties of congruent figures***

In the diagram, *QRST* ≅ *WXYZ*.

a. Find the value of *x*.

b. Find the value of *y*.



Solution

a. You know $\angle Q \cong \angle W$.

$m\angle Q =$ $m\angle W$

$65° =$ $(5x + 5)°$

$60 =$ $5x$

$12 = x$

b. You know $\overline{QR} \cong \overline{WX}$.

$QR =$ WX

$6 =$ $y - x$

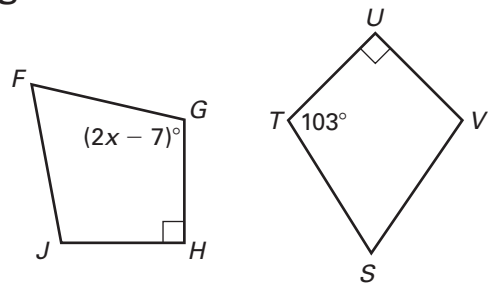
$6 =$ $y - 12$

$18 = y$

Your Notes

✔ ***Checkpoint*** **In Exercises 1 and 2, use the diagram shown in which *FGHJ* ≅ *STUV*.**

1. Identify all pairs of congruent corresponding parts.



Corresponding angles: $\angle F \cong \angle S$, $\angle G \cong \angle T$, $\angle H \cong \angle U$, $\angle J \cong \angle V$

Corresponding sides: $\overline{FG} \cong \overline{ST}$, $\overline{GH} \cong \overline{TU}$, $\overline{HJ} \cong \overline{UV}$, $\overline{JF} \cong \overline{VS}$

2. Find the value of x and find $m\angle G$.

$x = 55$; $m\angle G = 103°$

Example 3 ***Show that figures are congruent***

Maps If you cut the rectangular map in half along $\overline{PR}$, will the sections of the map be the same size and shape? *Explain*.

Solution

From the diagram, $\angle S \cong$ $\angle Q$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{PQ} \parallel$ $\overline{RS}$. Then $\angle 1 \cong$ $\angle 4$ and $\angle 2 \cong$ $\angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

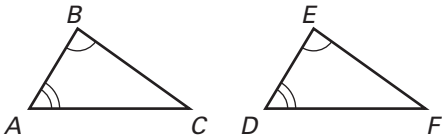
The diagram shows $\overline{PQ} \cong$ $\overline{RS}$ and $\overline{QR} \cong$ $\overline{SP}$. By the Reflexive Property, $\overline{PR} \cong \overline{RP}$. All corresponding parts are congruent, so $\triangle PQR \cong$ $\triangle RSP$.

Yes, the two sections will be the same size and shape.

Your Notes

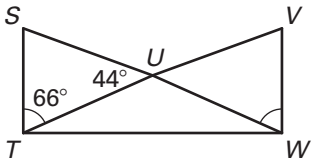
THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



Example 4 ***Use the Third Angles Theorem***

Find $m\angle V$.

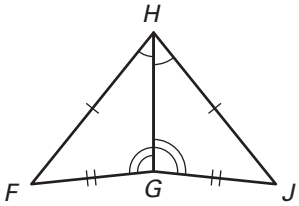


Solution

$\angle SUT \cong \angle VUW$ by the Vertical Angles Theorem. The diagram shows that $\angle STU \cong \angle VWU$, so by the Third Angles Theorem, $\angle S \cong \angle V$. By the Triangle Sum Theorem, $m\angle S = 180° - 66° - 44° = 70°$. So, $m\angle S = m\angle V = 70°$ by the definition of congruent angles.

Example 5 ***Prove that triangles are congruent***

Write a proof.



Given $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$, $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$

Prove $\triangle FGH \cong \triangle JGH$

Plan for Proof

a. Use the Reflexive Property to show $\overline{HG} \cong \overline{HG}$.

b. Use the Third Angles Theorem to show $\angle F \cong \angle J$.

Plan in Action

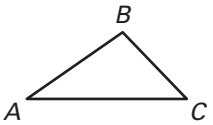
	Statements	Reasons
	1. $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$	1. Given
a.	2. $\overline{HG} \cong \overline{HG}$	2. Reflexive Property of Congruence
	3. $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$	3. Given
b.	4. $\angle F \cong \angle J$	4. Third Angles Theorem
	5. $\triangle FGH \cong \triangle JGH$	5. Definition of $\cong \triangle$s

Your Notes

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

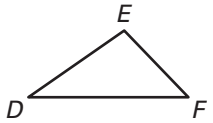
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong$ $\triangle ABC$.



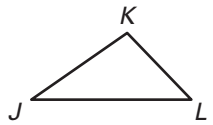
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



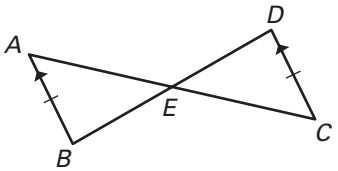
Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



✔ ***Checkpoint*** **Complete the following exercises.**

3. In the diagram at the right, E is the midpoint of $\overline{AC}$ and $\overline{BD}$. Show that $\triangle ABE \cong \triangle CDE$.

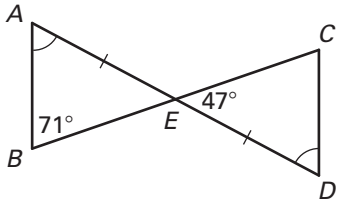


From the diagram, $\overline{AB} \cong \overline{CD}$. Point E is the midpoint of $\overline{AC}$ and $\overline{BD}$, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ by the definition of midpoint. So all pairs of corresponding sides are congruent.

The diagram shows $\overline{AB} \parallel \overline{CD}$, so $\angle A \cong \angle C$ and $\angle B \cong \angle D$ by the Alternate Interior Angles Theorem. Also, $\angle AEB \cong \angle CED$ by the Vertical Angles Theorem. All corresponding parts are congruent, so $\triangle ABE \cong \triangle CDE$.

4. In the diagram, what is the measure of $\angle D$?

62°



Homework

5. By the definition of congruence, what additional information is needed to know that $\triangle ABE \cong \triangle DCE$ in Exercise 4?

You must know that $\overline{AB} \cong \overline{DC}$ and $\overline{BE} \cong \overline{CE}$ to conclude that $\triangle ABE \cong \triangle DCE$. The remaining information can be inferred from the graph.

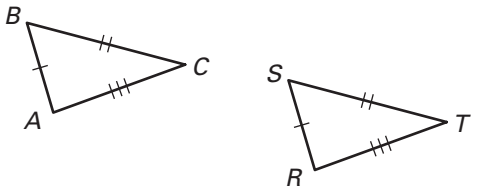
4.3 Prove Triangles Congruent by SSS

Goal • Use side lengths to prove triangles are congruent.

Your Notes

POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.



If Side $\overline{AB} \cong$ $\overline{RS}$,

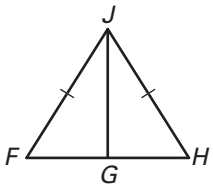
Side $\overline{BC} \cong$ $\overline{ST}$, and

Side $\overline{CA} \cong$ $\overline{TR}$,

then $\triangle ABC \cong$ $\triangle RST$.

Example 1 ***Use the SSS Congruence Postulate***

Write a proof.



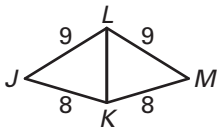
Given $\overline{FJ} \cong \overline{HJ}$,
G is the midpoint of $\overline{FH}$.

Prove $\triangle FGJ \cong \triangle HGJ$

Proof It is given that $\overline{FJ} \cong$ $\overline{HJ}$. Point G is the midpoint of $\overline{FH}$, so $\overline{FG} \cong \overline{HG}$. By the Reflexive Property, $\overline{GJ} \cong \overline{JG}$. So, by the SSS Congruence Postulate, $\triangle FGJ \cong \triangle HGJ$.

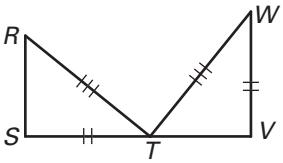
✔ ***Checkpoint*** **Decide whether the congruence statement is true. *Explain* your reasoning.**

1. $\triangle JKL \cong \triangle MKL$



True; all corresponding sides are congruent.

2. $\triangle RST \cong \triangle TVW$

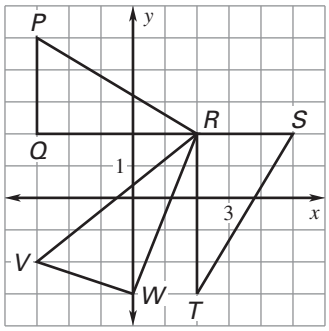


False; $\overline{RS} \ncong \overline{TV}$

Your Notes

Example 2 ***Congruence in the coordinate plane***

Determine whether $\triangle PQR$ is congruent to the other triangles shown at the right.



Solution

By counting, $PQ = 3$ and $QR = 5$. Use the distance formula to find PR.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

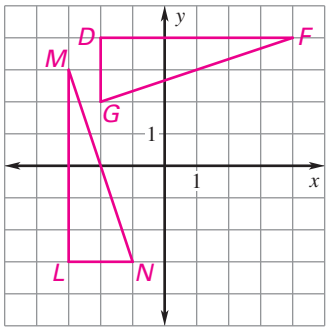
$PR = \sqrt{(2 - (-3))^2 + (2 - 5)^2} = \sqrt{34}$

By the SSS Congruence Postulate, any triangle with side lengths 3, 5, and $\sqrt{34}$ will be congruent to $\triangle PQR$. The distance from R to S is 3. The distance from R to T is 5. The distance from S to T is $\sqrt{(2 - 5)^2 + (-3 - 2)^2} = \sqrt{34}$. So, $\triangle PQR \cong \triangle SRT$.

The distance from W to V is $\sqrt{(-3 - 0)^2 + (-2 - (-3))^2} = \sqrt{10}$. No side of $\triangle PQR$ has a length of $\sqrt{10}$, so $\triangle PQR \ncong \triangle VWR$.

✔ ***Checkpoint*** **Complete the following exercise.**

3. $\triangle DFG$ has vertices $D(-2, 4)$, $F(4, 4)$, and $G(-2, 2)$. $\triangle LMN$ has vertices $L(-3, -3)$, $M(-3, 3)$, and $N(-1, -3)$. Graph the triangles in the same coordinate plane and show that they are congruent.

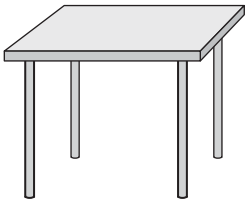


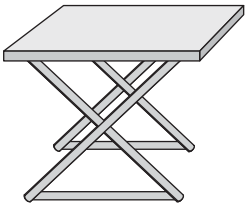
$DG = LN = 2$, $DF = LM = 6$, and $FG = MN = \sqrt{40}$, so $\triangle DFG \cong \triangle LMN$ by the SSS Congruence Postulate.

Your Notes

Example 3 ***Solve a real-world problem***

Stability *Explain* why the table with the diagonal legs is stable, while the one without the diagonal legs can collapse.



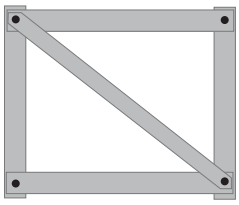


Solution

The table with the diagonal legs forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the table is stable. The table without the diagonal legs is not stable because there are many possible quadrilaterals with the given side lengths.

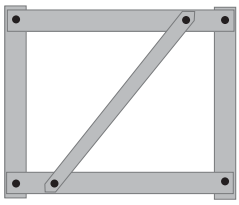
✔ ***Checkpoint*** **Determine whether the figure is stable. *Explain* your reasoning.**

4.



Yes, the figure is stable. By the SSS Congruence Postulate, the triangles formed cannot change shape, so it is stable.

5.



No, the figure is not stable. There are many possible quadrilaterals with the given side lengths.

Homework

4.4 Prove Triangles Congruent by SAS and HL

Goal • Use sides and angles to prove congruence.

Your Notes

VOCABULARY

Leg of a right triangle In a right triangle, a side adjacent to the right angle is called a leg.

Hypotenuse In a right triangle, the side opposite the right angle is called the hypotenuse.

POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong$ $\overline{UV}$,

Angle $\angle R \cong$ $\angle U$, and

Side $\overline{RT} \cong$ $\overline{UW}$,

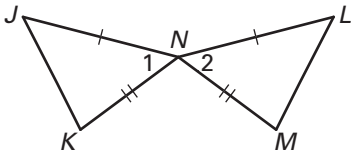
then $\triangle RST \cong$ $\triangle UVW$.

Example 1 ***Use the SAS Congruence Postulate***

Write a proof.

Given $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$

Prove $\triangle JKN \cong \triangle LMN$

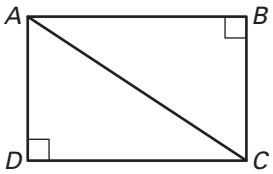


Statements	Reasons
1. $\overline{JN} \cong$ $\overline{LN}$, $\overline{KN} \cong$ $\overline{MN}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle JKN \cong \triangle LMN$	3. SAS Congruence Postulate

Your Notes

Example 2 ***Use SAS and properties of shapes***

In the diagram, *ABCD* is a rectangle. What can you conclude about △*ABC* and △*CDA*?

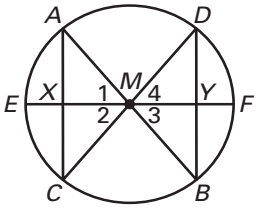


Solution

By the Right Angles Congruence Theorem, $\angle B \cong \angle D$. Opposite sides of a rectangle are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$.

△*ABC* and △*CDA* are congruent by the SAS Congruence Postulate.

✔ ***Checkpoint*** **In the diagram, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ pass through the center *M* of the circle. Also, $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.**



1. Prove that △*DMY* ≅ △*BMY*.

Statements	Reasons
1. $\angle 3 \cong \angle 4$	1. Given
2. $\overline{DM} \cong \overline{BM}$	2. Definition of a circle
3. $\overline{MY} \cong \overline{MY}$	3. Reflexive Property of Congruence
4. △*DMY* ≅ △*BMY*	4. SAS Congruence Postulate

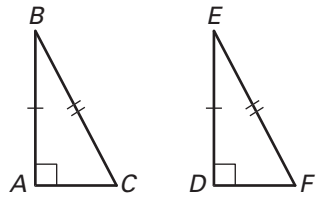
2. What can you conclude about $\overline{AC}$ and $\overline{BD}$?

Because they are vertical angles, $\angle AMC \cong \angle BMD$. All points on a circle are the same distance from the center, so $AM = BM = CM = DM$. By the SAS Congruence Postulate, △*AMC* ≅ △*BMD*. Corresponding parts of congruent triangles are congruent, so you know $\overline{AC} \cong \overline{BD}$.

Your Notes

THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are congruent.

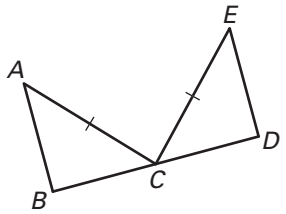


Example 3 ***Use the Hypotenuse-Leg Theorem***

Write a proof.

Given $\overline{AC} \cong \overline{EC}$, $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$, $\overline{AC}$ is a bisector of $\overline{BD}$.

Prove $\triangle ABC \cong \triangle EDC$

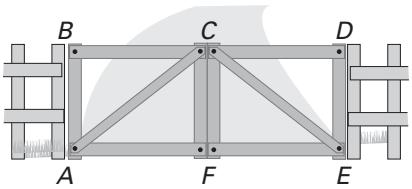


	Statements	Reasons
H	1. $\overline{AC} \cong \overline{EC}$	1. Given
	2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. Given
	3. $\angle B$ and $\angle D$ are right angles.	3. Definition of $\perp$ lines
	4. $\triangle ABC$ and $\triangle EDC$ are right triangles.	4. Definition of a right triangle
	5. $\overline{AC}$ is a bisector of $\overline{BD}$.	5. Given
L	6. $\overline{BC} \cong \overline{DC}$	6. Definition of segment bisector
	7. $\triangle ABC \cong \triangle EDC$	7. HL Congruence Theorem

Your Notes

Example 4 ***Choose a postulate or theorem***

Gate The entrance to a ranch has a rectangular gate as shown in the diagram. You know that $\triangle AFC \cong \triangle EFC$. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$?



Solution

You are given that *ABDE* is a rectangle, so $\angle B$ and $\angle D$ are right angles. Because opposite sides of a rectangle are congruent, $\overline{AB} \cong \overline{DE}$. You are also given that $\triangle AFC \cong \triangle EFC$, so $\overline{AC} \cong \overline{EC}$. The hypotenuse and a leg of each triangle is congruent.

You can use the HL Congruence Theorem to conclude that $\triangle ABC \cong \triangle EDC$.

✔ ***Checkpoint*** **Complete the following exercises.**

3. *Explain* why a diagonal of a rectangle forms a pair of congruent triangles.

A diagonal of a rectangle will be the hypotenuse of each triangle formed. Because the hypotenuse is congruent to itself, and because opposite sides of a rectangle are congruent, you can use the HL Congruence Theorem to conclude the triangles are congruent.

4. In Example 4, suppose it is given that *ABCF* and *EDCF* are squares. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$? *Explain*.

It is given that *ABCF* and *EDCF* are squares, so $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{DE}$, and $\overline{BC} \cong \overline{DC}$. You can use the SAS Congruence Postulate to conclude that $\triangle ABC \cong \triangle EDC$.

Homework

4.5 Prove Triangles Congruent by ASA and AAS

Goal • Use two more methods to prove congruences.

Your Notes

VOCABULARY

Flow proof A flow proof uses arrows to show the flow of a logical argument.

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

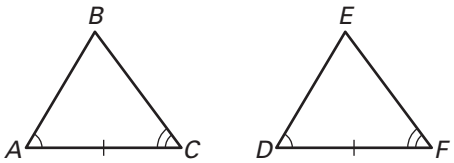
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If **Angle** $\angle A \cong$ $\angle D$,

Side $\overline{AC} \cong$ $\overline{DF}$, and

Angle $\angle C \cong$ $\angle F$,

then $\triangle ABC \cong$ $\triangle DEF$.



THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

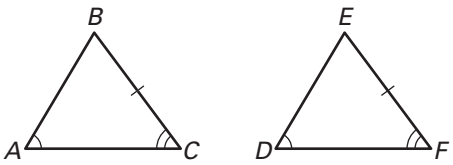
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If **Angle** $\angle A \cong$ $\angle D$,

Angle $\angle C \cong$ $\angle F$, and

Side $\overline{BC} \cong$ $\overline{EF}$,

then $\triangle ABC \cong$ $\triangle DEF$.

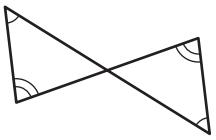


Your Notes

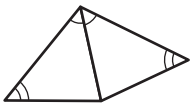
Example 1 ***Identify congruent triangles***

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

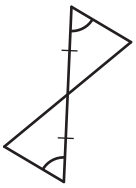
a.



b.



c.



Solution

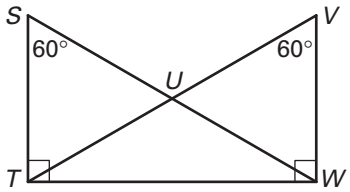
a. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

b. Two pairs of angles and a non-included pair of sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

c. The vertical angles are congruent, so two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

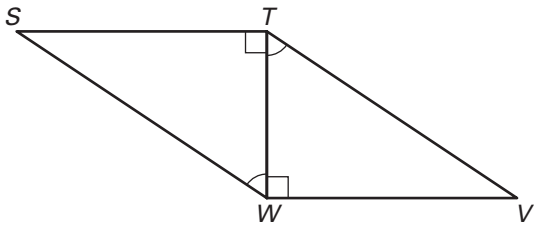
✔ ***Checkpoint*** **Can △*STW* and △*VWT* be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.**

1.



Yes; AAS Congruence Theorem

2.

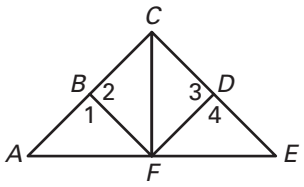


Yes; ASA Congruence Postulate

Your Notes

Example 2 ***Write a flow proof***

In the diagram, $\angle 1 \cong \angle 4$ and $\overline{CF}$ bisects $\angle ACE$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.



Solution

Given $\angle 1 \cong \angle 4$, $\overline{CF}$ bisects $\angle ACE$.

Prove $\triangle CBF \cong \triangle CDF$

$\angle 1 \cong \angle 4$ — Given

$\angle 1$ and $\angle 2$ are supplements.
$\angle 3$ and $\angle 4$ are supplements.
— Linear Pair Postulate

$\overline{CF}$ bisects $\angle ACE$. — Given

$\angle 2 \cong \angle 3$ — Congruent Supps. Thm.

$\overline{CF} \cong \overline{CF}$ — Reflexive Prop.

$\angle ACF \cong \angle ECF$ — Def. of $\angle$ bisector

$\angle CBF \cong \angle CDF$ — AAS Congruence Theorem

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

$\overline{CF}$ bisects $\angle ACE$. — Given

$\overline{CF}$ bisects $\angle BFD$. — Given

$\angle ACF \cong \angle ECF$ — Def. of $\angle$ bisector

$\overline{CF} \cong \overline{CF}$ — Reflexive Prop.

$\angle BFC \cong \angle DFC$ — Def. of $\angle$ bisector

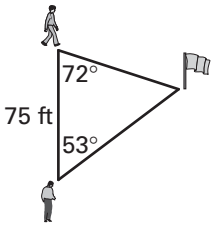
$\angle CBF \cong \angle CDF$ — ASA Congruence Postulate

Your Notes

Example 3 ***Choose a postulate or theorem***

Games You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is 72° and the angle from your friend to the flag is 53°. Is there enough information to locate the flag?

Solution

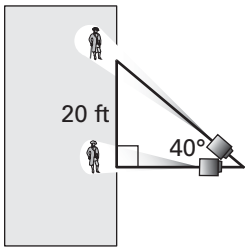


The locations of you, your friend, and the flag form a triangle. The measures of two angles and an included side of the triangle are known.

By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the flag location is given by the third vertex.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Theater Two actors are standing apart from each other on the edge of a stage. Spotlights are located and pointed as shown in the diagram. Can one of the actors move to another location on the stage without changing any of the angles of the triangle, without changing the distance to the other actor, and without requiring a spotlight to move?



No. The measures of two angles and a nonincluded side of the triangle are known. By the AAS Congruence Theorem, all triangles with these measures are congruent. To create a different congruent triangle, one of the actors would have to move to a location that is off the stage, and the spotlight following that actor would have to move.

Homework

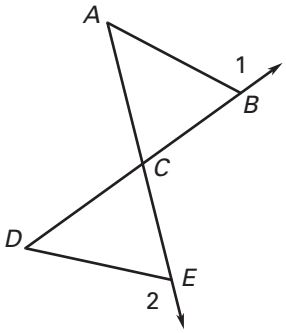
4.6 Use Congruent Triangles

Goal • Use congruent triangles to prove corresponding parts congruent.

Your Notes

Example 1 ***Use congruent triangles***

Explain **how you can use the given information to prove that the triangles are congruent.**



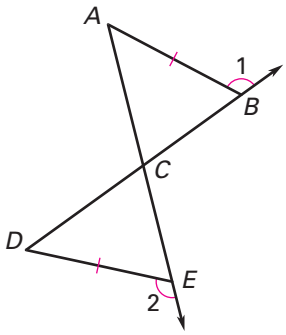
Given $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$

Prove $\overline{DC} \cong \overline{AC}$

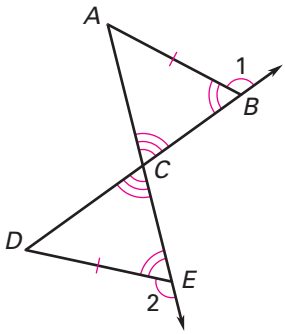
Solution

If you can show that $\triangle ABC \cong \triangle DEC$, you will know that $\overline{DC} \cong \overline{AC}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle ABC$ and $\angle DEC$ are supplementary to congruent angles, so $\angle ABC \cong \angle DEC$. Also, $\angle ACB \cong \angle DCE$.

Mark given information.



Add deduced information.



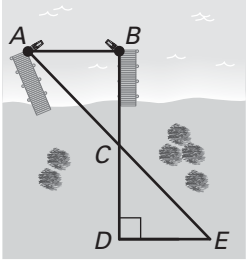
Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle ABC \cong \triangle DEC$. Because corresponding parts of congruent triangles are congruent, $\overline{DC} \cong \overline{AC}$.

Your Notes

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

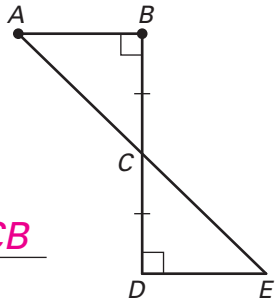
Example 2 *Use congruent triangles for measurement*

Boats Use the following method to find the distance between two docked boats, from point *A* to point *B*.



- Place a marker at *D* so that $\overline{AB} \perp \overline{BD}$.
- Find *C*, the midpoint of $\overline{BD}$.
- Locate the point *E* so that $\overline{BD} \perp \overline{DE}$ and *A*, *C*, and *E* are collinear.
- *Explain* how this plan allows you to find the distance.

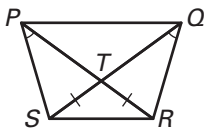
Solution



Because $\overline{AB} \perp \overline{BD}$ and $\overline{BD} \perp \overline{DE}$, $\angle B$ and $\angle D$ are congruent right angles. Because *C* is the midpoint of $\overline{BD}$, $\overline{BC} \cong \overline{DC}$. The vertical angles $\angle ACB$ and $\angle ECD$ are congruent. So, $\triangle CBA \cong \triangle CDE$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{BA} = \overline{DE}$. So, you can find the distance *AB* between the boats by measuring $\overline{DE}$.

✔ ***Checkpoint*** **Complete the following exercises.**

1. *Explain* how you can prove that $\overline{PR} \cong \overline{QS}$.



Use the AAS Congruence Theorem to show $\triangle PTS \cong \triangle QTR$. Because corresponding parts of congruent triangles are congruent, $\overline{PT} \cong \overline{QT}$. Then $\overline{PR} \cong \overline{QS}$ because $\overline{ST} \cong \overline{RT}$.

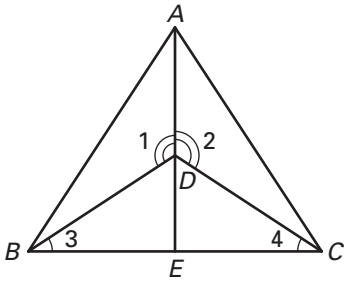
2. In Example 2, does it matter how far away from point *B* you place a marker at point *D*? *Explain*.

Point *D* should be placed far enough away from point *B* so that it is on land. This allows $\overline{DE}$ to be easily measured. However, the method will work regardless of how far *D* is from *B*.

Your Notes

Example 3 ***Plan a proof involving pairs of triangles***

Use the given information to write a plan for proof.



Given $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\triangle ABD \cong \triangle ACD$

Solution

In $\triangle ABD$ and $\triangle ACD$, you know that $\angle 1 \cong$ $\angle 2$ and $\overline{AD} \cong \overline{AD}$. If you can show that $\overline{BD} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

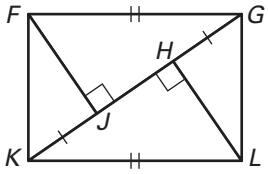
To prove that $\overline{BD} \cong \overline{CD}$, you can first prove that $\triangle BED \cong$ $\triangle CED$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property and $\angle BDE \cong$ $\angle CDE$ by the Congruent Supplements Theorem. You can use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle CED$.

Plan for Proof Use the AAS Congruence Theorem to prove that $\triangle BED \cong$ $\triangle CED$. Then state that $\overline{BD} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle ABD \cong \triangle ACD$.

✔ ***Checkpoint*** **Use the given information to write a plan for proof.**

3. Given $\overline{GH} \cong \overline{KJ}$, $\overline{FG} \cong \overline{LK}$, $\angle FJG$ and $\angle LHK$ are rt. $\angle$s.

Prove $\triangle FJK \cong \triangle LHG$

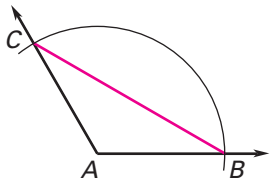


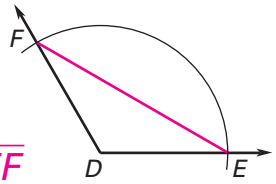
Plan for Proof: Use the HL Congruence Theorem to prove that $\triangle FJG \cong \triangle LHK$. Then state that $\overline{FJ} \cong \overline{LH}$. Then show that $\angle FJK \cong \angle LHG$ and use the SAS Congruence Postulate to prove that $\triangle FJK \cong \triangle LHG$.

Your Notes

Example 4 ***Justify a construction***

Write a proof to justify that the construction for copying an obtuse angle is valid.





Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.

Given $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

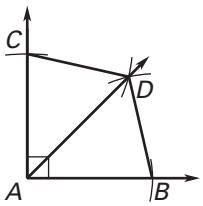
Prove $\angle D \cong \angle A$

Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle D$ and $\angle A$ are congruent.

	Statements	Reasons
Plan for Action	**1.** $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	**1.** Given
	2. $\triangle CAB \cong \triangle FDE$	**2.** SSS Congruence Postulate
	3. $\angle D \cong \angle A$	**3.** Corresp. parts of $\cong$ triangles are $\cong$.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Write a paragraph proof to verify that the construction for bisecting a right angle is valid.



You know that $\overline{AC} \cong \overline{AB}$ and $\overline{BD} \cong \overline{CD}$ because they are determined by the same compass settings. Also, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property. So, by the SSS Congruence Postulate, $\triangle CAD \cong \triangle BAD$. Thus, $\angle CAD \cong \angle BAD$ because corresponding parts of congruent triangles are congruent.

Homework

4.7 Use Isosceles and Equilateral Triangles

Goal • Use theorems about isosceles and equilateral triangles.

Your Notes

VOCABULARY

Legs The legs of an isosceles triangle are the two congruent sides.

Vertex angle The vertex angle of an isosceles triangle is the angle formed by the legs.

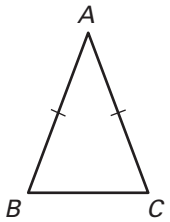
Base The base of an isosceles triangle is the side that is not a leg.

Base angles The base angles of an isosceles triangle are the two angles adjacent to the base.

THEOREM 4.7: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

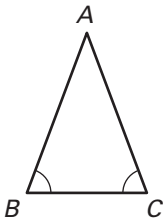
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong$ $\angle C$.



THEOREM 4.8: CONVERSE OF BASE ANGLES THEOREM

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong$ $\overline{AC}$.

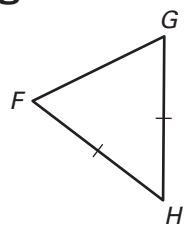


Example 1 ***Apply the Base Angles Theorem***

In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.

Solution

$\overline{FH} \cong \overline{GH}$, so by the Base Angles Theorem, $\angle$ F $\cong \angle$ G.



Your Notes

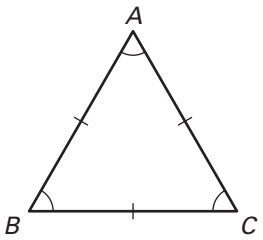
The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

COROLLARY TO THE BASE ANGLES THEOREM

If a triangle is equilateral, then it is equiangular.

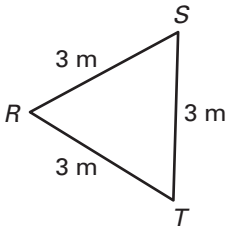
COROLLARY TO THE CONVERSE OF BASE ANGLES THEOREM

If a triangle is equiangular, then it is equilateral.



Example 2 ***Find measures in a triangle***

Find the measures of $\angle R$, $\angle S$, and $\angle T$.



Solution

The diagram shows that $\triangle RST$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle RST$ is equiangular. So, $m\angle R = m\angle S = m\angle T$.

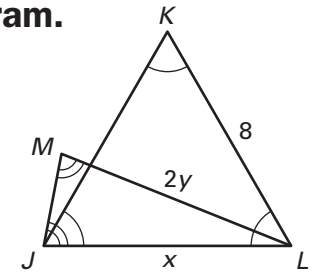
$3(m\angle R) =$ 180° **Triangle Sum Theorem**

$m\angle R =$ 60° **Divide each side by 3.**

The measures of $\angle R$, $\angle S$, and $\angle T$ are all 60°.

Example 3 ***Use isosceles and equilateral triangles***

Find the values of x and y in the diagram.



Solution

You cannot use $\angle J$ to refer to $\angle LJM$ because three angles have J as their vertex.

Step 1 **Find** the value of x. Because $\triangle JKL$ is equiangular, it is also equilateral and $\overline{KL} \cong \overline{JL}$. Therefore, $x =$ 8.

Step 2 Find the value of y. Because $\angle JML \cong \angle LJM$, $\overline{LM} \cong \overline{LJ}$, and $\triangle LMJ$ is isosceles. You know that $LJ =$ 8.

$LM = LJ$ **Definition of congruent segments**

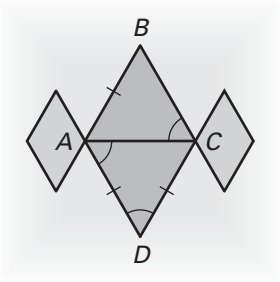
$2y =$ 8 **Substitute $2y$ for LM and 8 for LJ.**

$y =$ 4 **Divide each side by 2.**

Your Notes

Example 4 ***Solve a multi-step problem***

Quilting The pattern at the right is present in a quilt.



a. *Explain* why $\triangle ADC$ is equilateral.

b. Show that $\triangle CBA \cong \triangle ADC$.

Solution

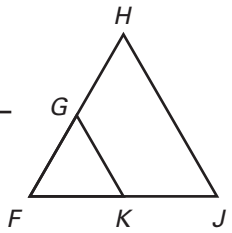
a. By the Base Angles Theorem, $\angle DAC \cong$ $\angle DCA$. So, $\triangle ADC$ is equiangular. By the Corollary to the Converse of Base Angles Theorem, $\triangle ADC$ is equilateral.

b. By the Base Angles Theorem, $\angle ABC \cong$ $\angle ACB$. So, $\triangle CBA \cong \triangle ADC$ by the AAS Congruence Theorem.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Copy and complete the statement: If $\overline{FH} \cong \overline{FJ}$, then $\angle$? $\cong \angle$?.

H; J



2. Copy and complete the statement: If $\triangle FGK$ is equiangular and $FG = 15$, then $GK =$?.

15

3. Use parts (a) and (b) in Example 4 to show that $m\angle BAD = 120°$.

$\triangle DCA$ is equiangular. So, $m\angle ADC = m\angle DCA = m\angle CAD$.

$3(m\angle CAD) = 180°$ Triangle Sum Theorem

$m\angle CAD = 60°$ Divide each side by 3.

Because $\triangle DCA$ is equiangular and $\triangle CBA \cong \triangle ADC$, you know that $m\angle BAC = 60°$.

$m\angle BAD = m\angle BAC + m\angle CAD$

$= 60° + 60°$

$= 120°$

Homework

4.8 Congruence Transformations and Coordinate Geometry

Goal • Create an image congruent to a given triangle.

Your Notes

VOCABULARY

Transformation A transformation is an operation that moves or changes a geometric figure in some way to produce a new figure.

Image The new figure produced by a transformation is the image.

Translation A translation moves every point of a figure the same distance in the same direction.

Reflection A reflection uses a *line of reflection* to create a mirror image of the original figure.

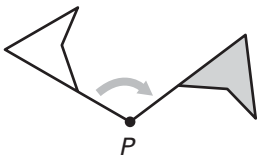
Rotation A rotation turns a figure about a fixed point, called the *center of rotation*.

Congruence Transformation A congruence transformation changes the position of a figure without changing its size or shape.

Example 1 ***Identify transformations***

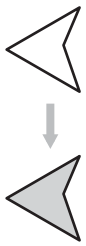
Name the type of transformation demonstrated in each picture.

a.



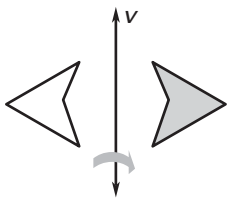
Rotation
about a point

b.



Translation
in a straight path

c.



Reflection
in a vertical line

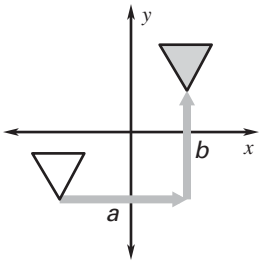
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



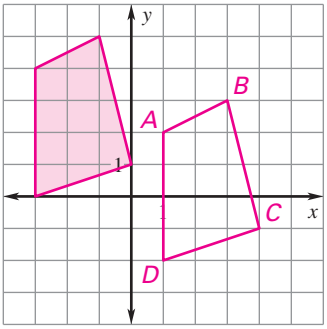
Example 2 ***Translate a figure in the coordinate plane***

Figure *ABCD* has the vertices *A*(1, 2), *B*(3, 3), *C*(4, −1), and *D*(1, −2). Sketch *ABCD* and its image after the translation $(x, y) \rightarrow (x - 4, y + 2)$.

Solution

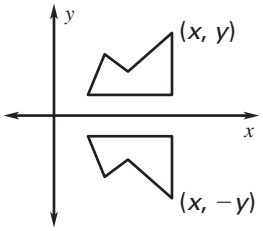
First draw *ABCD*. Find the translation of each vertex by subtracting 4 from its *x*-coordinate and adding 2 to its *y*-coordinate. Then draw *ABCD* and its image.

(x, y)	$\rightarrow$	$(x - 4, y + 2)$
$A(1, 2)$	$\rightarrow$	$(-3, 4)$
$B(3, 3)$	$\rightarrow$	$(-1, 5)$
$C(4, -1)$	$\rightarrow$	$(0, 1)$
$D(1, -2)$	$\rightarrow$	$(-3, 0)$



COORDINATE NOTATION FOR A REFLECTION

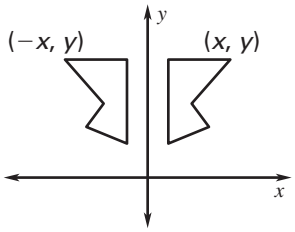
Reflection in the *x*-axis



Multiply *y*-coordinate by −1.

$(x, y) \rightarrow (x, -y)$

Reflection in the *y*-axis



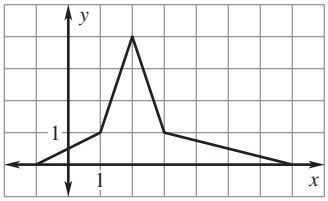
Multiply *x*-coordinate by −1.

$(x, y) \rightarrow (-x, y)$

Your Notes

Example 3 ***Reflect a figure in the x-axis***

Shapes You are cutting figures out of paper. Use a reflection in the *x*-axis to draw the other half of the figure.



Solution

Multiply the *y*-coordinate of each vertex by −1 to find the corresponding vertex in the image. Then draw the image.

$(x, y) \rightarrow$ $(x, -y)$

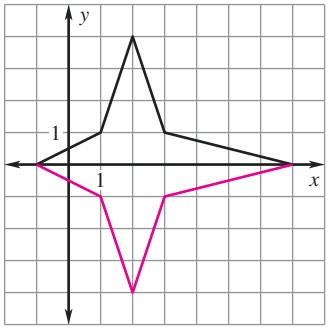
$(-1, 0) \rightarrow$ $(-1, 0)$

$(1, 1) \rightarrow$ $(1, -1)$

$(2, 4) \rightarrow$ $(2, -4)$

$(3, 1) \rightarrow$ $(3, -1)$

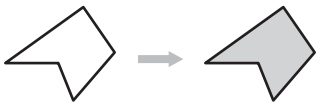
$(7, 0) \rightarrow$ $(7, 0)$



You can check your results by looking to see if each original point and its image are the same distance from the *x*-axis.

✔ ***Checkpoint*** **Complete the following exercises.**

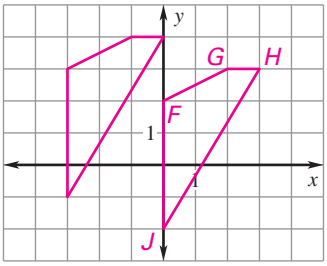
1. Name the type of transformation shown.



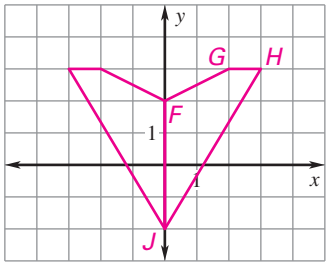
Translation

2. Figure *FGHJ* has the vertices *F*(0, 2), *G*(2, 3), *H*(3, 3), and *J*(0, −2). Sketch *FGHJ* and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the *y*-axis.

a.



b.



Your Notes

Example 4 ***Identify a rotation***

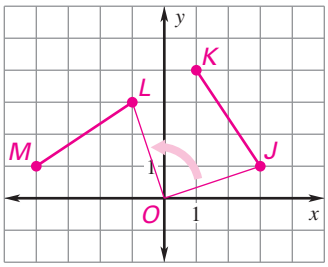
Graph $\overline{JK}$ and $\overline{LM}$. Tell whether $\overline{LM}$ is a rotation of $\overline{JK}$ about the origin. If so, give the angle and direction of rotation.

a. $J(3, 1)$, $K(1, 4)$, $L(-1, 3)$, $M(-4, 1)$

b. $J(-2, 1)$, $K(-1, 5)$, $L(1, 1)$, $M(2, 5)$

Solution

a.

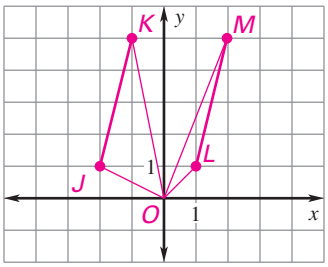


$m\angle JOL$ = $m\angle KOM$

= 90°

90° counterclockwise rotation

b.

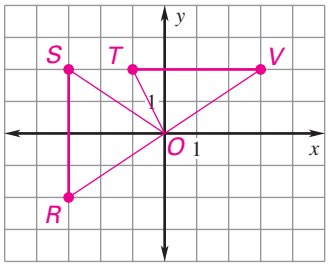


$m\angle JOL$ > $m\angle KOM$

not a rotation

✔ ***Checkpoint*** **Graph $\overline{RS}$ and $\overline{TV}$. Tell whether $\overline{TV}$ is a rotation of $\overline{RS}$ about the origin. If so, give the angle of rotation.**

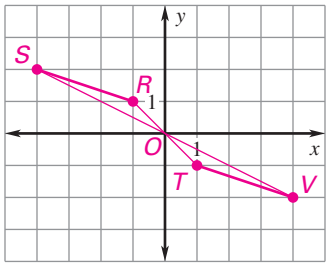
3. $R(-3, -2)$, $S(-3, 2)$, $T(-1, 2)$, $V(3, 2)$



$m\angle ROT < m\angle SOV$

not a rotation

4. $R(-1, 1)$, $S(-4, 2)$, $T(1, -1)$, $V(4, -2)$



$m\angle ROT = m\angle SOV$

= 180°

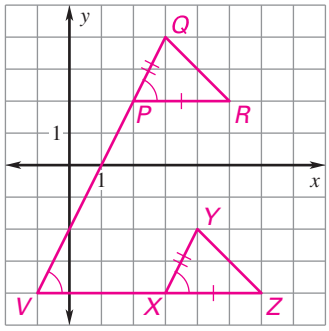
180° rotation

Your Notes

Example 5 ***Verify congruence***

The vertices of $\triangle PQR$ are $P(2, 2)$, $Q(3, 4)$, and $R(5, 2)$. The notation $(x, y) \rightarrow (x + 1, y - 6)$ describes the translation of $\triangle PQR$ to $\triangle XYZ$. Show that $\triangle PQR \cong \triangle XYZ$ to verify that the translation is a congruence transformation.

Solution



S You can see that $PR =$ XZ $=$ 3, so $\overline{PR} \cong$ $\overline{XZ}$.

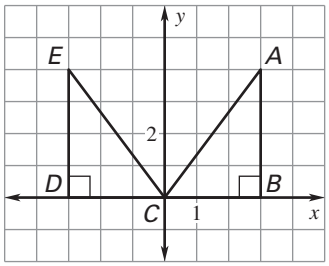
A Using the slopes, $\overline{PQ} \parallel$ $\overline{XY}$ and $\overline{QR} \parallel$ $\overline{YZ}$. If you extend $\overline{PQ}$ and $\overline{XZ}$ to form $\angle V$, the Corresponding Angles Postulate gives you $\angle QPR$ $\cong \angle V$ and $\angle V \cong$ $\angle YXZ$. Then, $\angle QPR$ $\cong$ $\angle YXZ$ by the Transitive Property of Congruence.

S Using the distance formula, $PQ =$ XY $=$ $\sqrt{5}$ so $\overline{PQ} \cong$ $\overline{XY}$. So, $\triangle PQR \cong \triangle XYZ$ by the SAS Congruence Postulate.

Because $\triangle PQR \cong \triangle XYZ$, the translation is a congruence transformation.

Checkpoint **Complete the following exercise.**

5. Show that $\triangle ABC \cong \triangle EDC$ to verify that the transformation is a congruence transformation.

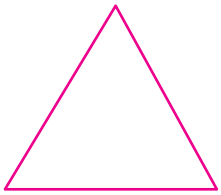
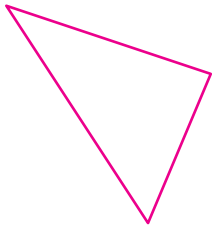
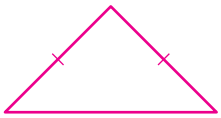
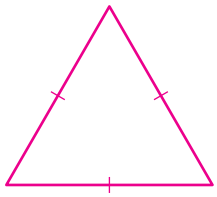
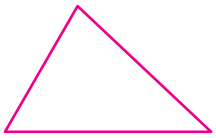
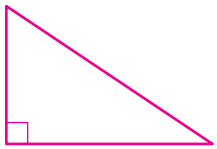
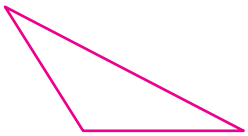
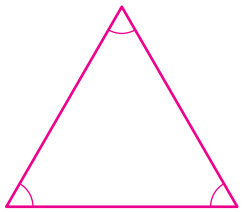


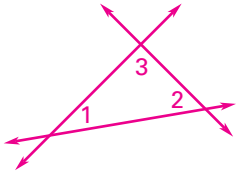
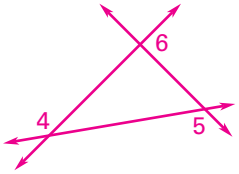
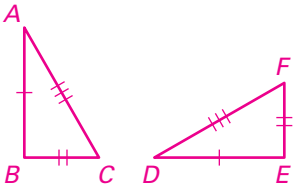
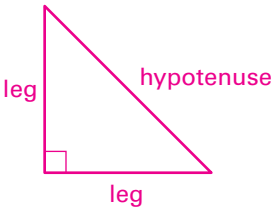
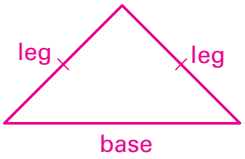
You can see that $AB = ED = 4$ and $BC = DC = 3$. Also, $\angle B$ and $\angle D$ are congruent right angles. So, $\triangle ABC \cong \triangle EDC$ by the SAS Congruence Postulate.

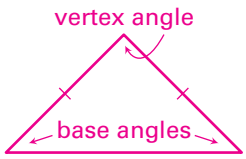
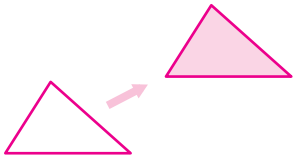
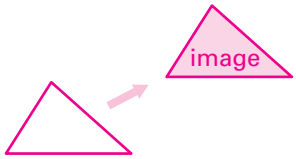
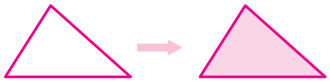
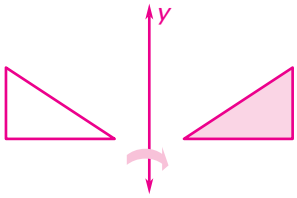
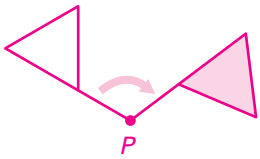
Homework

Words to Review

Give an example of the vocabulary word.

Triangle 	Scalene triangle 
Isosceles triangle 	Equilateral triangle 
Acute triangle 	Right triangle 
Obtuse triangle 	Equiangular triangle 

Interior angles  $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles.	**Exterior angles**  $\angle 4$, $\angle 5$, and $\angle 6$ are exterior angles.
Corollary to a theorem A corollary to a theorem is a statement that can be proved easily using the theorem.	**Congruent figures**  $\triangle ABC \cong \triangle DEF$
Corresponding parts If $\triangle ABC \cong \triangle DEF$, then $\angle A$ and $\angle D$ are corresponding parts.	**Leg of a right triangle, hypotenuse** 
Flow proof Statement 1 — Reason 1 Statement 2 — Reason 2 Statement 3 — Reason 3 Statement 4 — Reason 4	**Legs, base of an isosceles triangle** 

Vertex angle, base angles of an isosceles triangle  	Transformation 
Image  	Translation 
Reflection  	Rotation 
Congruence transformation Translations, reflections, and rotations are three types of congruence transformations. Because they change the position of a figure without changing its size or shape.	

Review your notes and Chapter 4 by using the Chapter Review on pages 295–298 of your textbook.

5.1 Midsegment Theorem and Coordinate Proof

Goal • Use properties of midsegments and write coordinate proofs.

Your Notes

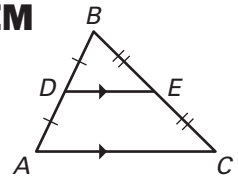
VOCABULARY

Midsegment of a triangle A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

Coordinate proof A coordinate proof involves placing geometric figures in a coordinate plane.

THEOREM 5.1: MIDSEGMENT THEOREM

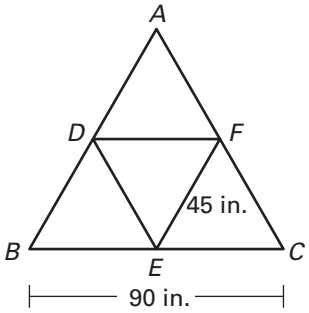
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.



$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

Example 1 ***Use the Midsegment Theorem to find lengths***

Windows A large triangular window is segmented as shown. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$. Find DF and AB.



In the diagram for Example 1, midsegment $\overline{DF}$ can be called "the midsegment opposite $\overline{BC}$."

Solution

$DF = \frac{1}{2} \cdot BC = \frac{1}{2}$ (90 in.) = 45 in.

$AB =$ 2 $\cdot FE =$ 2 (45 in.) = 90 in.

✔ ***Checkpoint*** **Complete the following exercise.**

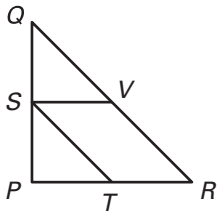
1. In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite $\overline{DF}$?

22.5 in.

Your Notes

Example 2 ***Use the Midsegment Theorem***

In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.



Solution

Because $QS = SP$ and $PT = TR$, S is the midpoint of $\overline{QP}$ and T is the midpoint of $\overline{PR}$ by definition. Then $\overline{ST}$ is a midsegment of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the Midsegment Theorem.

Checkpoint **Complete the following exercise.**

2. In Example 2, if V is the midpoint of $\overline{QR}$, what do you know about $\overline{SV}$?

$\overline{SV}$ is a midsegment of $\triangle PQR$ and $\overline{SV} \parallel \overline{PR}$.

Example 3 ***Place a figure in a coordinate plane***

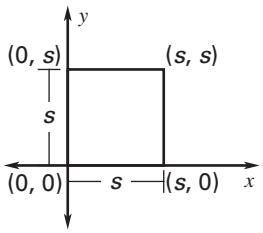
Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. a square **b.** an acute triangle

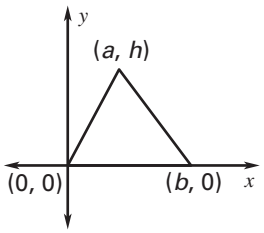
Solution

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let s represent the side length.



b. You need to use three different variables.



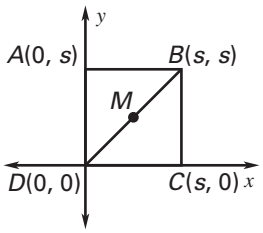
The square represents a general square because the coordinates are based only on the definition of a square. If you use this square to prove a result, the result will be true for all squares.

Your Notes

Example 4 ***Apply variable coordinates***

In Example 3 part (a), find the length and midpoint of a diagonal of the square.

Solution



Draw a diagonal and its midpoint. Assign letters to the points.

Use the distance formula to find *BD*.

$BD = \sqrt{(s-0)^2 + (s-0)^2}$

$= \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2}$

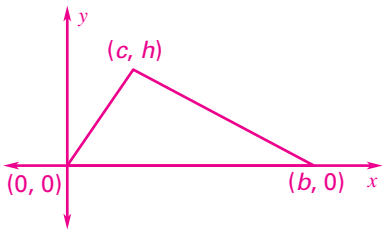
Use the midpoint formula to find the midpoint *M*.

$M\left(\frac{s+0}{2}, \frac{s+0}{2}\right) = M\left(\frac{s}{2}, \frac{s}{2}\right)$

✔ ***Checkpoint*** **Complete the following exercises.**

3. Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

Sample answer:



4. In Example 4, find the length and midpoint of diagonal $\overline{AC}$. What do you notice? *Explain* why this is true for all squares.

length: $s\sqrt{2}$; midpoint: $M\left(\frac{s}{2}, \frac{s}{2}\right)$; The lengths of the diagonals are the same, and the midpoints of the diagonals are the same. This is true for all squares because the coordinates are based only on the definition of a square.

Homework

5.2 Use Perpendicular Bisectors

Goal • Use perpendicular bisectors to solve problems.

Your Notes

VOCABULARY

Perpendicular bisector A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a perpendicular bisector.

Equidistant A point is equidistant from two figures if the point is the *same distance* from each figure.

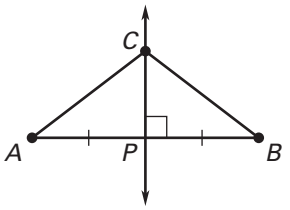
Concurrent When three or more lines, rays, or segments intersect in the same point, they are called concurrent lines, rays, or segments.

Point of concurrency The point of intersection of concurrent lines, rays, or segments is called the point of concurrency.

Circumcenter The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle.

THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM

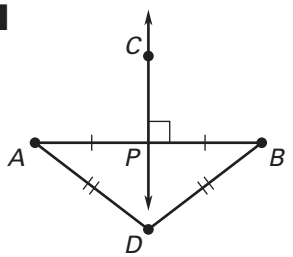
In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA =$ CB.

THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

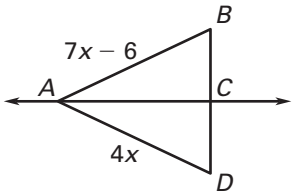


If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.

Your Notes

Example 1 ***Use the Perpendicular Bisector Theorem***

$\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find AD.



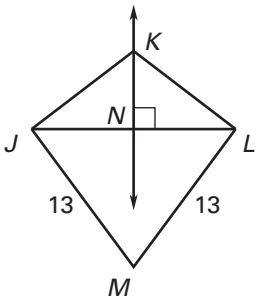
Solution

AD =	AB	**Perpendicular Bisector Theorem**
4x =	7x − 6	**Substitute.**
x =	2	**Solve for x.**

AD = 4x = 4(2) = 8 .

Example 2 ***Use perpendicular bisectors***

In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.



a. What segment lengths in the diagram are equal?

b. Is M on $\overleftrightarrow{KN}$?

Solution

a. $\overleftrightarrow{KN}$ bisects $\overline{JL}$, so NJ = NL . Because K is on the perpendicular bisector of $\overline{JL}$, KJ = KL by Theorem 5.2. The diagram shows that MJ = ML = 13.

b. Because $MJ = ML$, M is equidistant from J and L. So, by the Converse of the Perpendicular Bisector Theorem, M is on the perpendicular bisector of $\overline{JL}$, which is $\overleftrightarrow{KN}$.

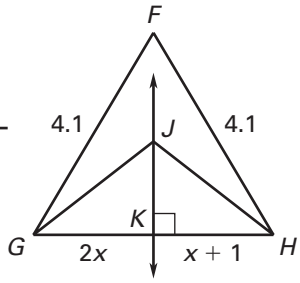
✔ ***Checkpoint*** **In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{GH}$.**

1. What segment lengths are equal?

$KG = KH$, $JG = JH$, $FG = FH$

2. Find GH.

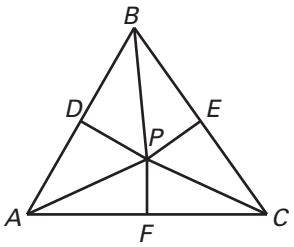
$GH = 4$



Your Notes

THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE

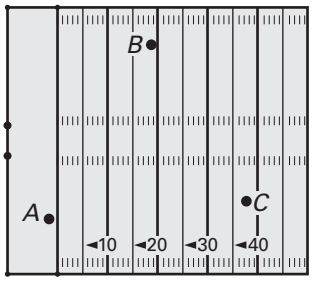
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.



If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then PA = PB = PC.

Example 3 ***Use the concurrency of perpendicular bisectors***

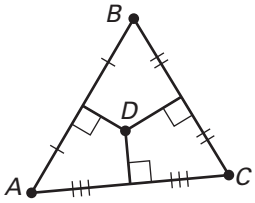
Football Three friends are playing catch. You want to join and position yourself so that you are the same distance from your friends. Find a location for you to stand.



Solution

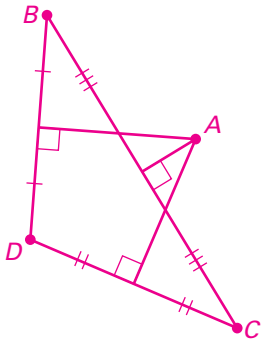
Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and a protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency D is a location for you to stand.



✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, your friend at location A wants to move to a location that is the same distance from everyone else. Find a new location for A.



Homework

5.3 Use Angle Bisectors of Triangles

Goal • Use angle bisectors to find distance relationships.

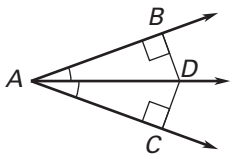
Your Notes

VOCABULARY

Incenter The point of concurrency of the three angle bisectors of a triangle is called the incenter of the triangle.

THEOREM 5.5: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

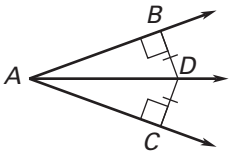


If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB =$ DC.

In geometry, *distance* means the *shortest* length between two objects.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.



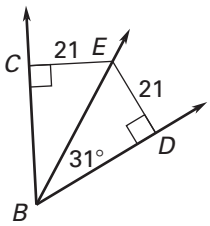
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

Example 1 ***Use the Angle Bisector Theorems***

Find the measure of $\angle CBE$.

Solution

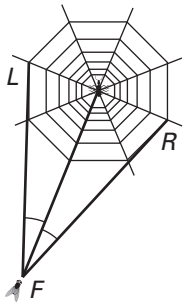
Because $\overline{EC} \perp \overrightarrow{BC}$ and $\overline{ED} \perp \overrightarrow{BD}$ and $EC = ED = 21$, $\overrightarrow{BE}$ bisects $\angle CBD$ by the Converse of the Angle Bisector Theorem. So, $m\angle CBE = m\angle DBE = 31°$.



Your Notes

Example 2 ***Solve a real-world problem***

Web A spider's position on its web relative to an approaching fly and the opposite sides of the web forms congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



Solution

The congruent angles tell you that the spider is on the bisector of $\angle LFR$. By the Angle Bisector Theorem, the spider is equidistant from $\overrightarrow{FL}$ and $\overrightarrow{FR}$.

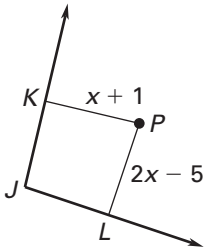
So, the spider must move the same distance to reach each edge.

Example 3 ***Use algebra to solve a problem***

For what value of x does P lie on the bisector of $\angle J$?

Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle J$ if P is equidistant from the sides of $\angle J$, so when $PK = PL$.



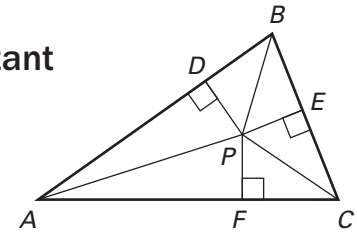
$PK = PL$	**Set segment lengths equal.**
$x + 1 = 2x - 5$	**Substitute expressions for segment lengths.**
$6 = x$	**Solve for x.**

Point P lies on the bisector of $\angle J$ when $x = 6$.

THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

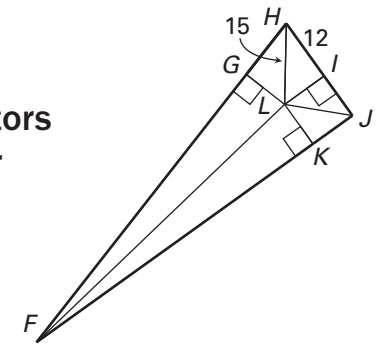
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



Your Notes

Example 4 ***Use the concurrency of angle bisectors***

In the diagram, *L* is the incenter of △*FHJ*. Find *LK*.



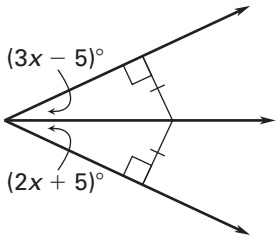
By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter *L* is equidistant from the sides of △*FHJ*. So, to find *LK*, you can find *LI* in △*LHI*. Use the Pythagorean Theorem.

c^2	$= a^2 + b^2$	**Pythagorean Theorem**
15^2	$= LI^2 + 12^2$	**Substitute known values.**
81	$= LI^2$	**Simplify.**
9	$= LI$	**Take the positive square root of each side.**

Because *LI* = *LK*, *LK* = 9.

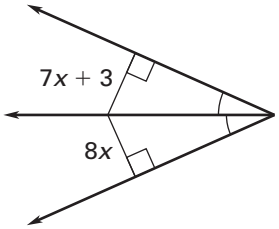
Checkpoint **In Exercises 1 and 2, find the value of *x*.**

1.



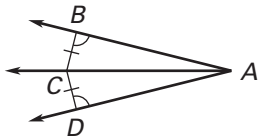
$x = 10$

2.



$x = 3$

3. Do you have enough information to conclude that $\overrightarrow{AC}$ bisects ∠*DAB*? *Explain*.



No, you must know that $m\angle ABC = m\angle ADC = 90°$ before you can conclude that $\overrightarrow{AC}$ bisects ∠*DAB*.

4. In Example 4, suppose you are not given *HL* or *HI*, but you are given that *JL* = 25 and *JI* = 20. Find *LK*.

LK = 15

Homework

5.4 Use Medians and Altitudes

Goal • Use medians and altitudes of triangles.

Your Notes

VOCABULARY

Median of a triangle The median of a triangle is a segment from a vertex to the midpoint of the opposite side.

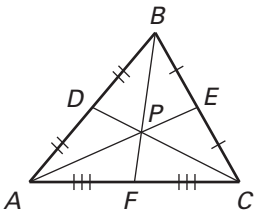
Centroid The point of concurrency of the three medians of a triangle is the centroid.

Altitude of a triangle An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

Orthocenter The point at which the lines containing the three altitudes of a triangle intersect is called the orthocenter of the triangle.

THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

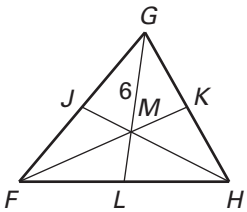


The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}$ AE, $BP = \frac{2}{3}$ BF, and $CP = \frac{2}{3}$ CD.

Your Notes

Example 1 ***Use the centroid of a triangle***

In △*FGH*, *M* is the centroid and *GM* = 6. Find *ML* and *GL*.



$\underline{GM} = \underline{\frac{2}{3}}\,GL$ **Concurrency of Medians of a Triangle Theorem**

$\underline{6} = \underline{\frac{2}{3}}\,GL$ **Substitute** $\underline{6}$ **for *GM*.**

$\underline{9} = GL$ **Multiply each side by the reciprocal,** $\underline{\frac{3}{2}}$.

Then $ML = GL - \underline{GM} = \underline{9} - \underline{6} = \underline{3}$.

So, $ML = \underline{3}$ and $GL = \underline{9}$.

✔ ***Checkpoint*** **Complete the following exercise.**

1. In Example 1, suppose *FM* = 10. Find *MK* and *FK*.

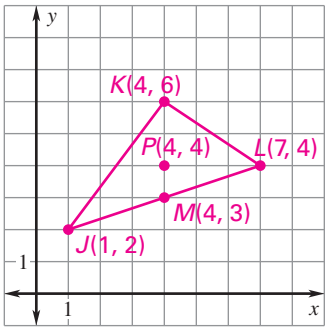
MK = 5, *FK* = 15

Example 2 ***Find the centroid of a triangle***

The vertices of △*JKL* are *J*(1, 2), *K*(4, 6), and *L*(7, 4). Find the coordinates of the centroid *P* of △*JKL*.

Sketch △*JKL*. Then use the Midpoint Formula to find the midpoint *M* of $\overline{JL}$ and sketch median $\overline{KM}$.

$$M\left(\frac{\boxed{1+7}}{2}, \frac{\boxed{2+4}}{2}\right) = \underline{M(4, 3)}$$



The centroid is $\underline{\text{two thirds}}$ of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex *K* to point *M* is $6 - \underline{3} = \underline{3}$ units. So, the centroid is $\underline{\frac{2}{3}}(\underline{3}) = \underline{2}$ units down from *K* on $\overline{KM}$.

The coordinates of the centroid *P* are $(4, 6 - \underline{2})$, or $(\underline{4, 4})$.

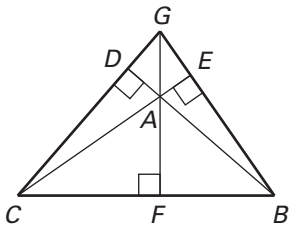
Median $\overline{KM}$ was used in Example 2 because it is easy to find distances on a vertical segment. You can check by finding the centroid using a different median.

Your Notes

THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

The lines containing the altitudes of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.

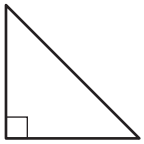


Example 3 ***Find the orthocenter***

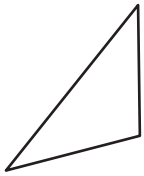
Notice that in a right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

Find the orthocenter *P* in the triangle.

a.

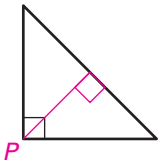


b.

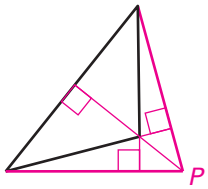


Solution

a.



b.

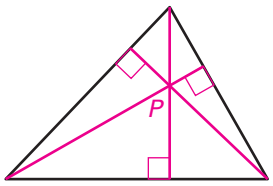


✔ ***Checkpoint*** **Complete the following exercises.**

2. In Example 2, where do you need to move point *K* so that the centroid is *P*(4, 5)?

Point *K* should be moved to (4, 9).

3. Find the orthocenter *P* in the triangle.

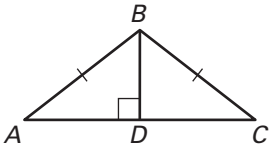


Your Notes

Example 4 ***Prove a property of isosceles triangles***

Prove that the altitude to the base of an isosceles triangle is a median.

Solution



Given $\triangle ABC$ is isosceles, with base $\overline{AC}$. $\overline{BD}$ is the altitude to base $\overline{AC}$.

Prove $\overline{BD}$ is a median of $\triangle ABC$.

Proof Legs $\overline{AB}$ and $\overline{CD}$ of $\triangle ABC$ are congruent. $\angle ADB$ and $\angle CDB$ are congruent right angles because $\overline{BD}$ is the altitude to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the HL Congruence Theorem.

$\overline{AD} \cong \overline{CD}$ because corresponding parts of congruent triangles are congruent. So, D is the midpoint of $\overline{AC}$ by definition. Therefore, $\overline{BD}$ intersects $\overline{AC}$ at its midpoint, and $\overline{BD}$ is a median of $\triangle ABC$.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Prove that the altitude $\overline{BD}$ in Example 4 is also an angle bisector.

Proof Legs $\overline{AB}$ and $\overline{CB}$ of $\triangle ABC$ are congruent. $\angle ADB$ and $\angle CDB$ are congruent right angles because $\overline{BD}$ is the altitude to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ADB \cong \triangle CDB$ by the HL Congruence Theorem. $\angle ABD \cong \angle CBD$ because corresponding parts of congruent triangles are congruent. Therefore, $\overline{BD}$ bisects $\angle ABC$, and $\overline{BD}$ is an angle bisector.

Homework

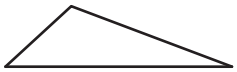
5.5 Use Inequalities in a Triangle

Goal • Find possible side lengths of a triangle.

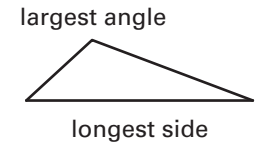
Your Notes

Example 1 ***Relate side length and angle measure***

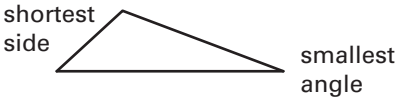
Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?



Solution



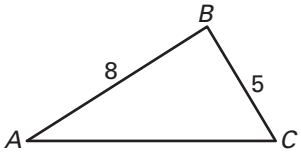
The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

THEOREM 5.10

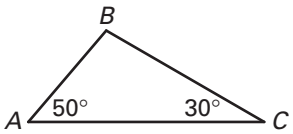
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



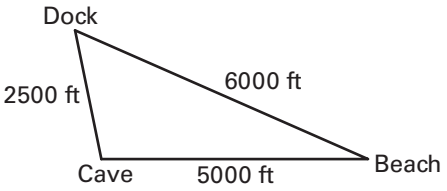
$m\angle A > m\angle C$, so $BC > AB$.

Be careful not to confuse the symbol $\angle$ meaning *angle* with the symbol $<$ meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

Your Notes

Example 2 ***Find angle measures***

Boating A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about 55° and one is about 24°. What is the angle measure of the path made at the cave?



Solution

The cave is opposite the longest side so, by Theorem 5.10, the cave angle is the largest angle.

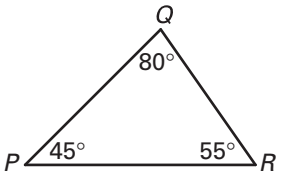
The angle measures sum to 180°, so the third angle measure is $180° - (55° + 24°)$ = 101°.

The angle measure made at the cave is 101°.

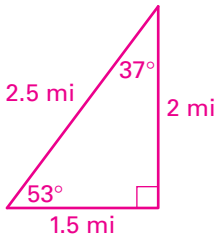
✔ ***Checkpoint*** **Complete the following exercises.**

1. List the sides of $\triangle PQR$ in order from shortest to longest.

$\overline{QR}$, $\overline{PQ}$, $\overline{PR}$



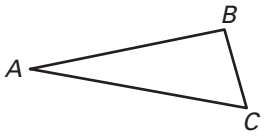
2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.



Your Notes

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



AB + BC > AC

AC + BC > AB

AB + AC > BC

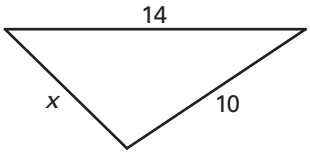
Example 3 ***Find possible side lengths***

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

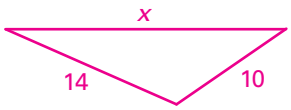
Small values of x



$x + 10 > 14$

$x > 4$

Large values of x



$10 + 14 > x$

$24 > x$, or $x < 24$

The length of the third side must be greater than 4 and less than 24.

✔ ***Checkpoint*** **Complete the following exercise.**

Homework

3. A triangle has one side of 23 meters and another of 17 meters. *Describe* the possible lengths of the third side.

The length of the third side must be greater than 6 meters and less than 40 meters.

5.6 Inequalities in Two Triangles and Indirect Proof

Goal • Use inequalities to make comparisons in two triangles.

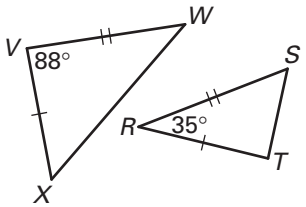
Your Notes

VOCABULARY

Indirect Proof An indirect proof uses a temporary assumption that the desired conclusion is false. By then showing that this assumption leads to a logical impossibility, the original statement is proven true *by contradiction*.

THEOREM 5.13: HINGE THEOREM

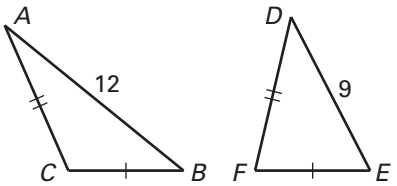
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.



$WX >$ ST

THEOREM 5.14: CONVERSE OF THE HINGE THEOREM

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

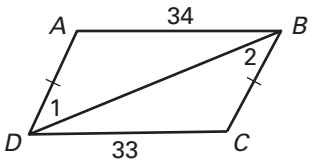


$m\angle C > m\angle$ F

Your Notes

Example 1 ***Use the Converse of the Hinge Theorem***

Given that $\overline{AD} \cong \overline{BC}$, how does $\angle 1$ compare to $\angle 2$?



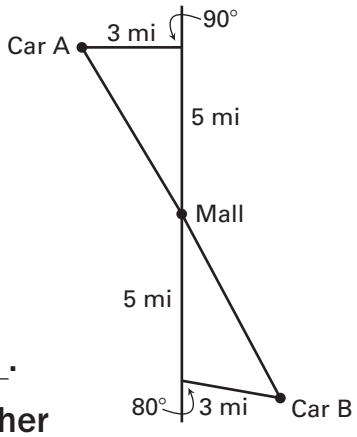
Solution

You are given that $\overline{AD} \cong \overline{BC}$ and you know that $\overline{BD} \cong \overline{BD}$ by the Reflexive Property. Because 34 > 33, AB > CD. So, two sides of $\triangle ADB$ are congruent to two sides of $\triangle CBD$ and the third side in $\triangle ADB$ is longer.

By the Converse of the Hinge Theorem, $m\angle 1 > m\angle 2$.

Example 2 ***Solve a multi-step problem***

Travel Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall?



Draw a diagram. The distance driven and the distance back to the mall form two triangles, with congruent 5 mile sides and congruent 3 mile sides. Add the third side to the diagram.

Use linear pairs to find the included angles of 90° and 100°.

Because 100° > 90°, Car B is farther from the mall than Car A by the Hinge Theorem.

HOW TO WRITE AN INDIRECT (Proof by Contradiction)

Step 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that the opposite is true.

Step 2 **Reason** logically until you reach a contradiction.

Step 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Your Notes

Example 3 ***Write an indirect proof***

Write an indirect proof to show that an odd number is not divisible by 6.

Given x is an odd number.

Prove x is not divisible by 6.

Solution

Step 1 Assume temporarily that x is divisible by 6. This means that $\frac{x}{6} = n$ for some whole number n. So, multiplying both sides by 6 gives $x = 6n$.

You have reached a contradiction when you have two statements that cannot both be true at the same time.

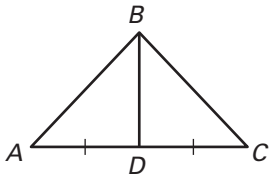
Step 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 6n$ so $\frac{x}{2} = \frac{6n}{2} = 3n$. We know that $3n$ is a whole number because n is a whole number, so x can be divided evenly by 2. This contradicts the given statement that x is odd.

Step 3 Therefore, the assumption that x is divisible by 6 is false, which proves that x is not divisible by 6.

✔ ***Checkpoint*** **Complete the following exercises.**

1. If $m\angle ADB > m\angle CDB$ which is longer, $\overline{AB}$ or $\overline{CB}$?

$\overline{AB}$ is longer.



2. In Example 2, car C leaves the mall and goes 5 miles due west, then turns 85° toward south for 3 miles. Write the cars in order from the car closest to the mall to the car farthest from the mall.

car A, car C, car B

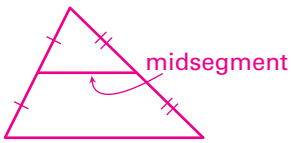
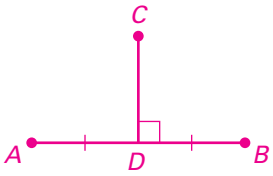
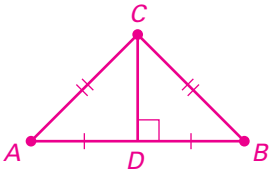
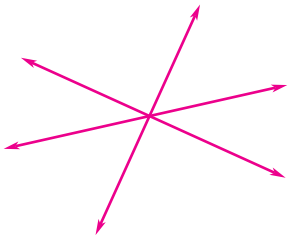
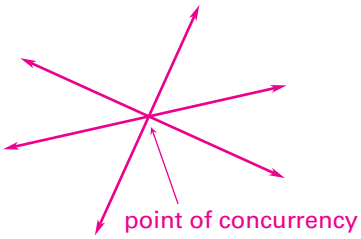
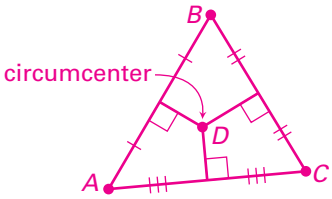
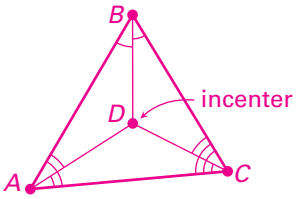
Homework

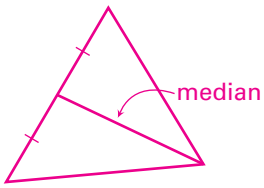
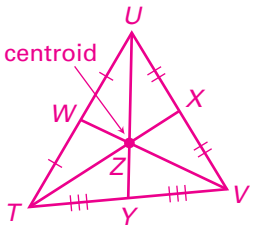
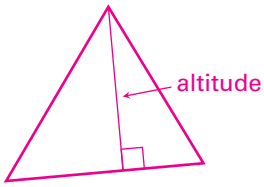
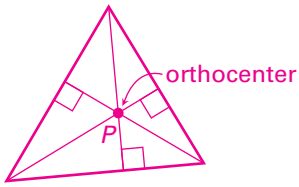
3. Suppose you wanted to prove the statement "If $x + y \neq 5$ and $y = 2$, then $x \neq 3$." What temporary assumption could you make to prove the conclusion indirectly?

You can temporarily assume that $x = 3$.

Words to Review

Give an example of the vocabulary word.

Midsegment of a triangle  	Coordinate proof A type of proof that involves placing geometric figures in a coordinate plane.
Perpendicular bisector  $\overline{CD}$ is a perpendicular bisector of $\overline{AB}$.	Equidistant  Point C is equidistant from point A and point B.
Concurrent  The lines are concurrent.	Point of concurrency 
Circumcenter  	Incenter 

Median of a triangle	Centroid
	
Altitude of a triangle	**Orthocenter**
	

Indirect proof

To use an indirect proof, assume that the original statement is false and that the opposite is true. If this assumption leads to a contradiction, then the assumption must be false and the original statement must be true.

Review your notes and Chapter 5 by using the Chapter Review on pages 359–362 of your textbook.

6.1 Ratios, Proportions, and the Geometric Mean

Goal • Solve problems by writing and solving proportions.

Your Notes

VOCABULARY

Ratio of a to b If a and b are two numbers or quantities and $b \neq 0$, then the ratio of a to b is $\frac{a}{b}$.

Proportion An equation that states that two ratios are equal is a proportion.

Means, extremes In the proportion $\frac{a}{b} = \frac{c}{d}$, b and c are the means, and a and d are the extremes.

Geometric mean The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

Example 1 ***Simplify ratios***

Simplify the ratio. ***(See Table of Measures, p. 921)***

a. 76 cm : 8 cm **b.** $\frac{4 \text{ ft}}{24 \text{ in.}}$

Solution

a. Write 76 cm : 8 cm as $\frac{76 \text{ cm}}{8 \text{ cm}}$. Then divide out the units and simplify.

For help with conversion factors, see p. 992.

$$\frac{76 \cancel{\text{cm}}}{8 \cancel{\text{cm}}} = \frac{19}{2} = 19 : 2$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{4 \text{ ft}}{24 \text{ in.}} = \frac{4 \cancel{\text{ft}}}{24 \cancel{\text{in.}}} \cdot \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} = \frac{48}{24} = \frac{2}{1}$$

Your Notes

Example 2 ***Use a ratio to find a dimension***

Painting You are painting barn doors. You know that the perimeter of the doors is 64 feet and that the ratio of the length to the height is 3 : 5. Find the area of the doors.

Solution

Step 1 **Write** expressions for the length and height. Because the ratio of the length to height is 3 : 5, you can represent the length by 3 x and the height by 5 x.

Step 2 **Solve** an equation to find x.

$2\ell + 2w = P$	**Formula for perimeter**
$2(3x) + 2(5x) = 64$	**Substitute.**
$16x = 64$	**Multiply and combine like terms.**
$x = 4$	**Divide each side by 16.**

Step 3 **Evaluate** the expressions for the length and height. Substitute the value of x into each expression.

Length: $3x = 3(4) = 12$

Height: $5x = 5(4) = 20$

The doors are 12 feet long and 20 feet high, so the area is $12 \cdot 20 = 240\ \text{ft}^2$.

✔ ***Checkpoint*** **In Exercises 1 and 2, simplify the ratio.**

1. 4 meters to 18 meters 2 to 9	**2.** 33 yd : 9 ft 11 : 1

3. The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5 : 2. Find the length and width of the table.

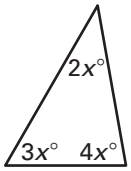
length: 7.5 feet, width: 3 feet

Your Notes

Example 3 *Use extended ratios*

The measures of the angles in △*BCD* are in the *extended ratio* of 2 : 3 : 4. Find the measures of the angles.

Solution



Begin by sketching the triangle. Then use the extended ratio of 2 : 3 : 4 to label the measures as 2 $x°$, 3 $x°$, and 4 $x°$.

2 $x°$ + 3 $x°$ + 4 $x°$ = 180°	**Triangle Sum Theorem**
9 x = 180	**Combine like terms.**
x = 20	**Divide each side by 9 .**

The angle measures are 2(20°) = 40°, 3(20°) = 60°, and 4(20°) = 80°.

✔ ***Checkpoint*** **Complete the following exercise.**

4. A triangle's angle measures are in the extended ratio of 1 : 4 : 5. Find the measures of the angles.

18°, 72°, 90°

A PROPERTY OF PROPORTIONS

1. Cross Products Property In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$\frac{2}{3} = \frac{4}{6}$ 3 • 4 = 12

2 • 6 = 12

Your Notes

In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{3}{4} = 16 \cdot \frac{x}{16}$ so $\underline{12} = x$.

Example 4 ***Solve proportions***

Solve the proportion.

a. $\frac{3}{4} = \frac{x}{16}$ **Original proportion**

$3 \cdot \underline{16} = \underline{4} \cdot x$ **Cross Products Property**

$\underline{48} = \underline{4}\,x$ **Multiply.**

$\underline{12} = x$ **Divide each side by** $\underline{4}$**.**

b. $\frac{3}{x+1} = \frac{2}{x}$ **Original proportion**

$\underline{3} \cdot x = \underline{2}(x + 1)$ **Cross Products Property**

$\underline{3}\,x = \underline{2}\,x + \underline{2}$ **Distributive Property**

$x = \underline{2}$ **Subtract** $\underline{2x}$ **from each side.**

Example 5 ***Solve a real-world problem***

Bowling You want to find the total number of rows of boards that make up 24 lanes at a bowling alley. You know that there are 117 rows in 3 lanes. Find the total number of rows of boards that make up the 24 lanes.

Solution

Write and solve a proportion involving two ratios that compare the number of rows with the number of lanes.

$\frac{\underline{117}}{\underline{3}} = \frac{\underline{n}}{\underline{24}}$ ← number of rows ← number of lanes **Write proportion.**

$\underline{117} \cdot \underline{24} = \underline{3} \cdot \underline{n}$ **Cross Products Property**

$\underline{936} = n$ **Simplify.**

There are $\underline{936}$ rows of boards that make up the 24 lanes.

GEOMETRIC MEAN

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

So, $x^2 = \underline{ab}$ and $x = \sqrt{\underline{ab}}$.

Your Notes

Example 6 ***Find a geometric mean***

Find the geometric mean of 16 and 48.

Solution

$x = \sqrt{ab}$	**Definition of geometric mean**
$= \sqrt{16 \cdot 48}$	**Substitute** 16 **for** a **and** 48 **for** b.
$= \sqrt{16 \cdot 16 \cdot 3}$	**Factor.**
$= 16\sqrt{3}$	**Simplify.**

The geometric mean of 16 and 48 is $16\sqrt{3} \approx 27.7$.

✔ ***Checkpoint*** **Complete the following exercises.**

5. Solve $\frac{8}{y} = \frac{2}{5}$.

$y = 20$

6. Solve $\frac{x - 3}{3} = \frac{2x}{9}$.

$x = 9$

7. A small gymnasium contains 10 sets of bleachers. You count 192 spectators in 3 sets of bleachers and the spectators seem to be evenly distributed. Estimate the total number of spectators.

about 640 spectators

8. Find the geometric mean of 14 and 16.

$4\sqrt{14} \approx 15.0$

Homework

6.2 Use Proportions to Solve Geometry Problems

Goal • Use proportions to solve geometry problems.

Your Notes

VOCABULARY

Scale drawing A scale drawing is a drawing that is the same shape as the object it represents.

Scale The scale is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

ADDITIONAL PROPERTIES OF PROPORTIONS

2. Reciprocal Property If two ratios are equal, then their reciprocals are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

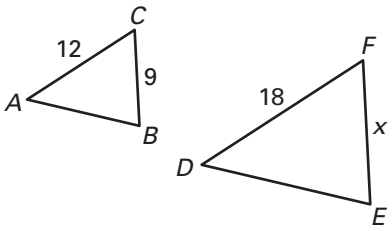
4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

Your Notes

Example 1 ***Use properties of proportions***

In the diagram, $\frac{AC}{DF} = \frac{BC}{EF}$. Write four true proportions.



Because $\frac{AC}{DF} = \frac{BC}{EF}$, then $\frac{12}{18} = \frac{9}{x}$.

Reciprocal Property: The reciprocals are equal, so $\frac{18}{12} = \frac{x}{9}$.

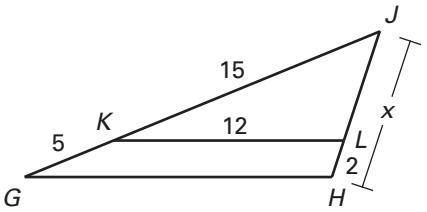
Property 3: You can interchange the means, so $\frac{12}{9} = \frac{18}{x}$.

Property 4: You can add the denominators to the numerators, so $\frac{30}{18} = \frac{9 + x}{x}$.

Example 2 ***Use proportions with geometric figures***

In the diagram, $\frac{JL}{LH} = \frac{JK}{KG}$.

Find *JH* and *JL*.



$\frac{JL}{LH} = \frac{JK}{KG}$	**Given**
$\frac{JL + LH}{LH} = \frac{JK + KG}{KG}$	**Property of Proportions (Property 4)**
$\frac{x}{2} = \frac{15 + 5}{5}$	**Substitution Property of Equality**
$5x = 2(15 + 5)$	**Cross Products Property**
$x = 8$	**Solve for *x*.**

So $JH = 8$ and $JL = 8 - 2 = 6$.

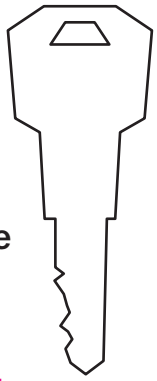
✔ ***Checkpoint*** **Complete the following exercises.**

1. In Example 1, find the value of *x*. $x = 13.5$	**2.** In Example 2, $\frac{KL}{GH} = \frac{JK}{JG}$. Find *GH*. $GH = 16$

Your Notes

Example 3 ***Find the scale of a drawing***

Keys The length of the key in the scale drawing is 7 centimeters. The length of the actual key is 4 centimeters. What is the scale of the drawing?



Solution

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length in drawing}}{\boxed{\text{length of key}}} = \frac{7\text{ cm}}{4\text{ cm}} = \frac{7 \div 4}{4 \div 4} = \frac{1.75}{1}$$

The scale of the drawing is 1.75 cm : 1 cm.

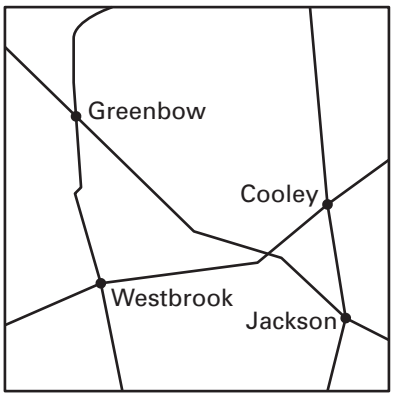
✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose the length of the key in the scale drawing is 6 centimeters. Find the new scale of the drawing.

1.5 cm : 1 cm

Example 4 ***Use a scale drawing***

Maps The scale of the map at the right is 1 inch : 8 miles. Find the actual distance from Westbrook to Cooley.



Solution

Use a ruler. The distance from Westbrook to Cooley on the map is about 1.25 inches. Let x be the actual distance in miles.

$$\frac{1.25\text{ in.}}{x\text{ mi}} = \frac{1\text{ in.}}{8\text{ mi}} \begin{array}{l} \leftarrow \text{distance on map} \\ \leftarrow \text{actual distance} \end{array}$$

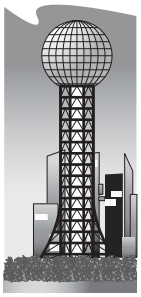
$x = 1.25(8)$ **Cross Products Property**

$x = 10$ **Simplify.**

The actual distance from Westbrook to Cooley is about 10 miles.

Your Notes

Example 5 ***Solve a multi-step problem***



Scale Model You buy a 3-D scale model of the Sunsphere in Knoxville, TN. The actual building is 266 feet tall. Your model is 20 inches tall, and the diameter of the dome on your scale model is about 5.6 inches.

a. What is the diameter of the actual dome?

b. How many times as tall as your model is the actual building?

Solution

a. $\frac{20 \text{ in.}}{266 \text{ ft}} = \frac{5.6 \text{ in.}}{x \text{ ft}}$ ← measurement on model ← measurement on actual building

20 x = 1489.6 **Cross Products Property**

$x \approx$ 74.5 **Divide each side by** 20 **.**

The diameter of the actual dome is about 74.5 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$\frac{266 \text{ ft}}{20 \text{ in.}} = \frac{266 \text{ ft}}{20 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} =$ 159.6

The actual building is 159.6 times as tall as the model.

✔ ***Checkpoint*** **Complete the following exercises.**

4. Two landmarks are 130 miles from each other. The landmarks are 6.5 inches apart on a map. Find the scale of the map.

1 inch : 20 miles

5. Your friend has a model of the Sunsphere that is 5 inches tall. What is the approximate diameter of the dome on your friend's model?

about 1.4 inches

Homework

6.3 Use Similar Polygons

Goal • Use proportions to identify similar polygons.

Your Notes

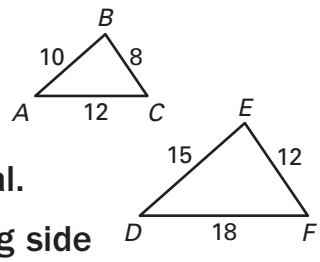
VOCABULARY

Similar polygons Two polygons are similar polygons if corresponding angles are congruent and corresponding side lengths are proportional.

Scale factor of two similar polygons If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the scale factor.

Example 1 ***Use similarity statements***

In the diagram, $\triangle ABC \sim \triangle DEF$.



a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle A \cong \angle$ D, $\angle B \cong \angle$ E, $\angle C \cong \angle$ F

b. $\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$ $\quad \frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$

$\frac{CA}{FD} = \frac{12}{18} = \frac{2}{3}$

c. The ratios in part (b) are equal, so

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

✔ ***Checkpoint*** **Complete the following exercise.**

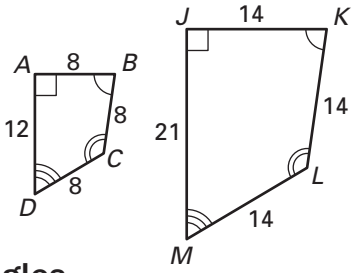
1. Given $\triangle PQR \sim \triangle XYZ$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

$\angle P \cong \angle X$, $\angle Q \cong \angle Y$, $\angle R \cong \angle Z$; $\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX}$

Your Notes

Example 2 ***Find the scale factor***

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ABCD* to *JKLM*.



Solution

Step 1 **Identify** pairs of congruent angles.
From the diagram, you can see that $\angle B \cong \angle$ K, $\angle C \cong \angle$ L, and $\angle D \cong \angle$ M. Angles A and J are right angles, so $\angle$ A $\cong \angle$ J.
So, the corresponding angles are congruent.

Step 2 **Show** that corresponding side lengths are proportional.

$\frac{AB}{JK} = \frac{8}{14} = \frac{4}{7}$ $\qquad$ $\frac{BC}{KL} = \frac{8}{14} = \frac{4}{7}$

$\frac{CD}{LM} = \frac{8}{14} = \frac{4}{7}$ $\qquad$ $\frac{AD}{JM} = \frac{12}{21} = \frac{4}{7}$

The ratios are equal, so the corresponding side lengths are proportional.

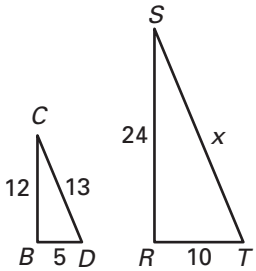
So *ABCD* ~ *JKLM*. The scale factor of *ABCD* to *JKLM* is $\frac{4}{7}$.

Example 3 ***Use similar polygons***

In the diagram, $\triangle BCD \sim \triangle RST$. Find the value of *x*.

Solution

The triangles are similar, so the corresponding side lengths are proportional.



$\frac{BC}{\boxed{RS}} = \frac{\boxed{CD}}{ST}$ **Write proportion.**

$\frac{12}{\boxed{24}} = \frac{\boxed{13}}{x}$ **Substitute.**

$12x = 312$ **Cross Products Property**

$x = 26$ **Solve for *x*.**

There are several ways to write the proportion. For example, you could write $\frac{BD}{RT} = \frac{CD}{ST}$.

Your Notes

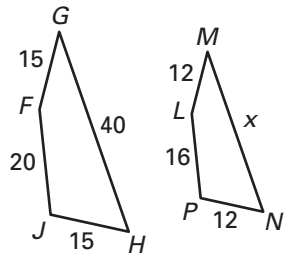
✔ ***Checkpoint*** **In the diagram, *FGHJ* ~ *LMNP*.**

2. What is the scale factor of *LMNP* to *FGHJ*?

$\frac{4}{5}$

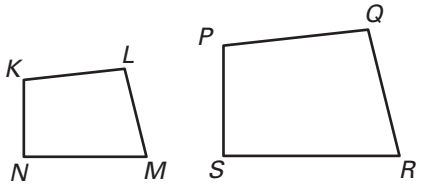
3. Find the value of *x*.

32



THEOREM 6.1: PERIMETERS OF SIMILAR POLYGONS

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If *KLMN* ~ *PQRS*, then

$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}.$$

Example 4 ***Find perimeters of similar figures***

Basketball A larger cement court is being poured for a basketball hoop in place of a smaller one. The court will be 20 feet wide and 25 feet long. The old court was similar in shape, but only 16 feet wide.

a. Find the scale factor of the new court to the old court.

b. Find the perimeters of the new court and the old court.

Solution

a. Because the new court will be similar to the old court, the scale factor is the ratio of the widths, $\frac{20}{16} = \frac{5}{4}$.

b. The new court's perimeter is $2(20) + 2(25) = 90$ feet. Use Theorem 6.1 to find the perimeter *x* of the old court.

$\frac{90}{x} = \frac{5}{4}$ **Use Theorem 6.1 to write a proportion.**

$x = 72$ **Simplify.**

The perimeter of the old court was 72 feet.

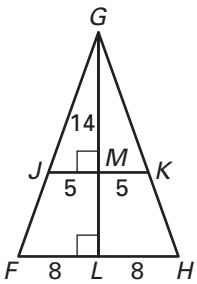
Your Notes

CORRESPONDING LENGTHS IN SIMILAR POLYGONS

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

Example 5 *Use a scale factor*

In the diagram, $\triangle FGH \sim \triangle JGK$. Find the length of the altitude $\overline{GL}$.



Solution

First, find the scale factor of $\triangle FGH$ to $\triangle JGK$.

$$\frac{FH}{JK} = \frac{8+8}{5+5} = \frac{16}{10} = \frac{8}{5}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{GL}{GM} = \frac{8}{5}$ **Write proportion.**

$\frac{GL}{14} = \frac{8}{5}$ **Substitute 14 for *GM*.**

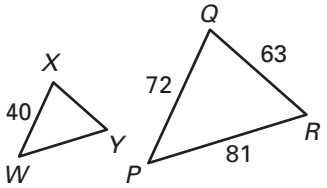
$GL = 22.4$ **Multiply each side by 14 and simplify.**

The length of altitude $\overline{GL}$ is 22.4.

Checkpoint **In the diagrams, $\triangle PQR \sim \triangle WXY$.**

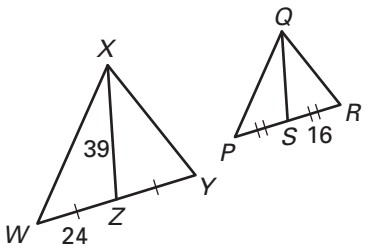
4. Find the perimeter of $\triangle WXY$.

The perimeter of $\triangle WXY$ is 120.



5. Find the length of median $\overline{QS}$.

$QS = 26$



Homework

Focus On Patterns

Use after Lesson 6.3

Your Notes

Fibonacci Sequence and the Golden Ratio

Goal • Use the Fibonacci sequence and the golden ratio.

VOCABULARY

Fibonacci sequence The sequence 1, 1, 2, 3, 5, ..., which uses the rule that each term after the second term is the sum of the previous two terms

Golden ratio The irrational number 1.618... approached by the ratios of consecutive terms from the Fibonacci sequence.

Golden rectangle A rectangle whose sides form a golden ratio.

Example 1 ***Find terms in the Fibonacci sequence***

Find the eighth and ninth terms of the Fibonacci sequence.

Solution

Each term of the Fibonacci sequence after the second term is the sum **of the previoue** two **terms.**

3rd term	**1 + 1 = 2**	**Sum of 1st and 2nd terms**
4th term	**1 + 2 = 3**	**Sum of 2nd and 3rd terms**
5th term	**2 + 3 = 5**	**Sum of 3rd and 4th terms**
6th term	**3 + 5 = 8**	**Sum of 4th and 5th terms**
7th term	**5 + 8 = 13**	**Sum of 5th and 6th terms**
8th term	**8 + 13 = 21**	**Sum of 6th and 7th terms**
9th term	**13 + 21 = 34**	**Sum of 7th and 8th terms**

✔ ***Checkpoint*** **Find terms of the Fibonacci sequence.**

1. What are the tenth and eleventh terms of the Fibonacci sequence?

55; 89

Your Notes

Example 2 ***Find ratios of terms in the Fibonacci sequence***

Find the ratios of consecutive terms in the Fibonacci sequence using the seventh, eighth, and ninth terms. Round to the nearest 0.001.

Solution

The seventh, eighth, and ninth terms of the Fibonacci sequence are 13, 21, **and** 34.

$$\frac{\text{8th term}}{\boxed{\text{7th term}}} = \frac{\boxed{21}}{13} = 1.615 \qquad \frac{\boxed{\text{9th term}}}{\boxed{\text{8th term}}} = \frac{\boxed{34}}{\boxed{21}} = 1.619$$

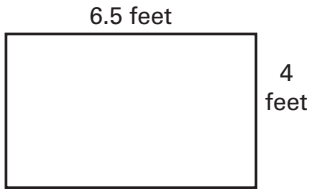
✔ ***Checkpoint*** **Complete the following exercise.**

2. Which two consecutive terms of the Fibonacci sequence will give you a ratio of approximately 1.618?

ninth and tenth terms

Example 3 ***The golden rectangle***

Show that the figure is nearly a golden rectangle.



Solution

For a rectangle to be a golden rectangle,

$$\frac{\text{length}}{\text{width}} = \frac{\boxed{\text{width}} + \boxed{\text{length}}}{\text{length}} \approx 1.618.$$

For the figure shown,

$$\frac{6.5}{\boxed{4}} = 1.625 \text{, and } \frac{\boxed{6.5} + \boxed{4}}{6.5} \approx 1.615.$$

Yes, **the figure** is **nearly a golden rectangle.**

Homework

✔ ***Checkpoint*** **Complete the following exercise.**

3. Is a 15 m × 13 m rectangle nearly a golden rectangle?

No

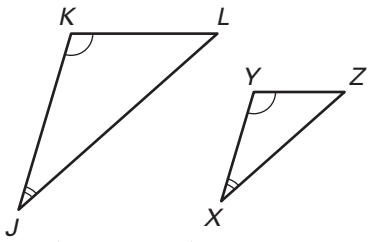
6.4 Prove Triangles Similar by AA

Goal • Use the AA Similarity Postulate.

Your Notes

POSTULATE 22: ANGLE-ANGLE (AA) SIMILARITY POSTULATE

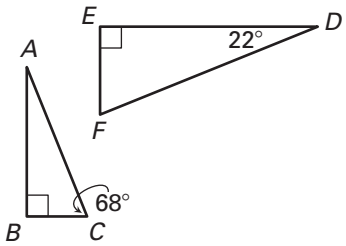
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

Example 1 ***Use the AA Similarity Postulate***

Determine whether the triangles are similar. If they are, write a similarity statement. *Explain* your reasoning.



Solution

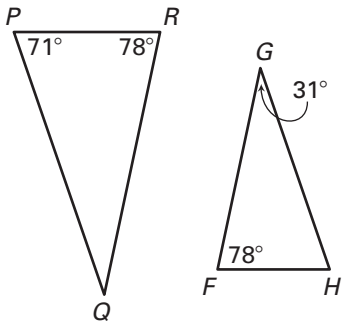
Because they are both right angles, $\angle$ B and $\angle$ E are congruent.

By the Triangle Sum Theorem,
68° + 90° + $m\angle A$ = 180°, so $m\angle A$ = 22°.
Therefore, $\angle A$ and $\angle$ D are congruent.

So, $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate.

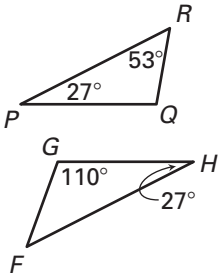
✔ ***Checkpoint*** **Determine whether the triangles are similar. If they are, write a similarity statement.**

1.



$\triangle FGH \sim \triangle RQP$

2.



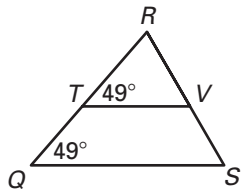
not similar

Your Notes

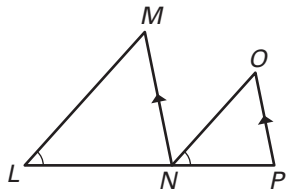
Example 2 ***Show that triangles are similar***

Show that the two triangles are similar.

a. $\triangle RTV$ and $\triangle RQS$



b. $\triangle LMN$ and $\triangle NOP$



Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle$ RTV and $m\angle$ Q both equal 49°, $\angle$ RTV $\cong \angle$ Q. By the Reflexive Property, $\angle R \cong \angle$ R.

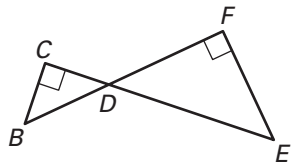
So, $\triangle RTV \sim \triangle RQS$ by the AA Similarity Postulate.

b. The diagram shows $\angle L \cong \angle$ ONP. It also shows that $\overline{MN} \parallel$ $\overline{OP}$ so $\angle$ LNM $\cong \angle$ P by the Corresponding Angles Postulate.

So, $\triangle LMN \sim \triangle NOP$ by the AA Similarity Postulate.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Show that $\triangle BCD \sim \triangle EFD$.



Because they are both right angles, $\angle C \cong \angle F$.

You know that $\angle CDB \cong \angle FDE$ by the Vertical Angles Congruence Theorem.

So, $\triangle BCD \sim \triangle EFD$ by the AA Similarity Postulate.

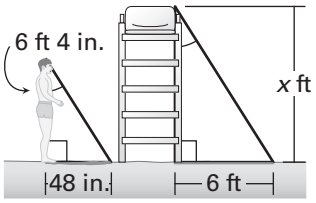
Your Notes

Example 3 ***Using similar triangles***

Height A lifeguard is standing beside the lifeguard chair on a beach. The lifeguard is 6 feet 4 inches tall and casts a shadow that is 48 inches long. The chair casts a shadow that is 6 feet long. How tall is the chair?

Solution

The lifeguard and the chair form sides of two right triangles with the ground, as shown below. The sun's rays hit the lifeguard and the chair at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 6 feet 4 inches as 76 inches so you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{76 \text{ in.}} = \frac{6 \text{ ft}}{48 \text{ in.}}$$ **Write proportion of side lengths.**

$$48x = 456$$ **Cross Products Property**

$$x = 9.5$$ **Solve for x.**

The chair is 9.5 feet tall.

✔ ***Checkpoint*** **Complete the following exercise.**

4. In Example 3, how long is the shadow of a person that is 4 feet 9 inches tall?

3 feet

Homework

6.5 Prove Triangles Similar by SSS and SAS

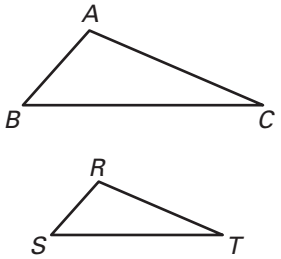
Goal • Use the SSS and SAS Similarity Theorems.

Your Notes

THEOREM 6.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

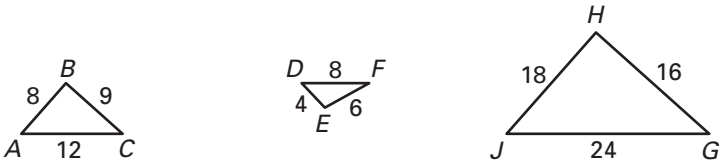
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.



Example 1 ***Use the SSS Similarity Theorem***

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Shortest sides	Longest sides	Remaining sides
$\frac{AB}{DE} = \frac{8}{4} = 2$	$\frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}$	$\frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}$

The ratios are not all equal, so $\triangle ABC$ and $\triangle DEF$ are not similar.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

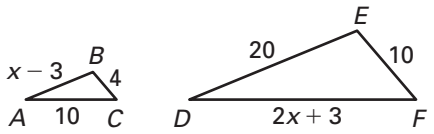
Shortest sides	Longest sides	Remaining sides
$\frac{AB}{GH} = \frac{8}{16} = \frac{1}{2}$	$\frac{CA}{JG} = \frac{12}{24} = \frac{1}{2}$	$\frac{BC}{HJ} = \frac{9}{18} = \frac{1}{2}$

All the ratios are equal, so $\triangle ABC \sim \triangle GHJ$.

Your Notes

Example 2 ***Use the SSS Similarity Theorem***

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Solution

Step 1 **Find** the value of x that makes corresponding side lengths proportional.

$$\frac{4}{\boxed{10}} = \frac{x-3}{\boxed{20}}$$ **Write proportion.**

$$4 \cdot \underline{20} = \underline{10}(x-3)$$ **Cross Products Property**

$$\underline{80} = \underline{10}x - \underline{30}$$ **Simplify.**

$$\underline{11} = x$$ **Solve for x.**

Step 2 **Check** that the side lengths are proportional when $x =$ 11.

$AB = x - 3 =$ 8 $\qquad DF = 2x + 3 =$ 25

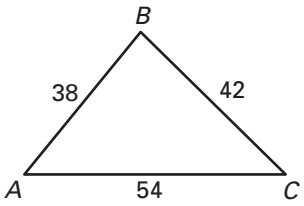
$\frac{BC}{EF} \stackrel{?}{=} \frac{AB}{DE}$ $\quad \frac{4}{10} = \frac{8}{20}$ ✓ $\qquad \frac{BC}{EF} \stackrel{?}{=} \frac{AC}{DF}$ $\quad \frac{4}{10} = \frac{10}{25}$ ✓

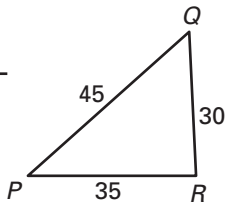
When $x =$ 11, the triangles are similar by the SSS Similarity Theorem.

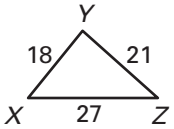
✔ ***Checkpoint*** **Complete the following exercises.**

1. Which of the three triangles are similar?

$\triangle PQR \sim \triangle ZXY$







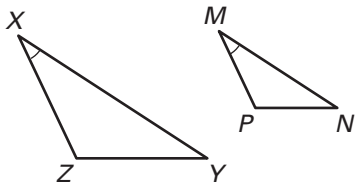
2. Suppose AB is not given in $\triangle ABC$. What value of AB would make $\triangle ABC$ similar to $\triangle QRP$?

$AB = 36$

Your Notes

THEOREM 6.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

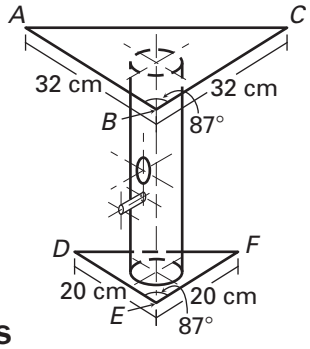
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



If $\angle X \cong \angle M$, and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Example 3 ***Use the SAS Similarity Theorem***

Birdfeeder You are drawing a design for a birdfeeder. Can you construct the top so it is similar to the bottom using the angle measure and lengths shown?



Solution

Both $m\angle$ B and $m\angle$ E equal 87°, so $\angle$ B $\cong \angle$ E. Next, compare the ratios of the lengths of the sides that include $\angle B$ and $\angle E$.

$$\frac{AB}{\boxed{DE}} = \frac{\boxed{BC}}{EF} = \frac{32}{20} = \frac{8}{5}$$

The lengths of the sides that include $\angle B$ and $\angle E$ are proportional.

So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle DEF$. Yes, you can make the top similar to the bottom.

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose you use equilateral triangles on the top and bottom. Are the top and bottom similar? *Explain*.

Yes, the top and bottom are similar. If the side length of the top is *a* and the side length of the bottom is *b*, the ratios of the side lengths are $\frac{a}{b}$ and the angles are all 60°. The triangles are similar by SAS or SSS.

Your Notes

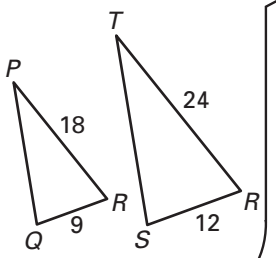
TRIANGLE SIMILARITY POSTULATE AND THEOREMS

AA Similarity Postulate If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

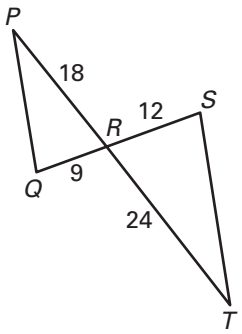
SAS Similarity Theorem If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Example 4 ***Choose a method***

Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

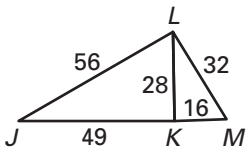
Shorter sides $\frac{QR}{RS} = \frac{9}{12} = \frac{3}{4}$

Longer sides $\frac{PR}{RT} = \frac{18}{24} = \frac{3}{4}$

The corresponding side lengths are proportional. The included angles $\angle PRQ$ and $\angle TRS$ are congruent because they are vertical angles. So, $\triangle PQR \sim \triangle TSR$ by the SAS Similarity Theorem.

✔ ***Checkpoint*** **Complete the following exercise.**

4. *Explain* how to show $\triangle JKL \sim \triangle LKM$.



Show that the corresponding side lengths are proportional, then use the SSS Similarity Theorem to show $\triangle JKL \sim \triangle LKM$.

Homework

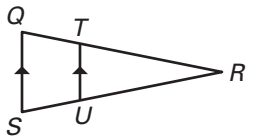
6.6 Use Proportionality Theorems

Goal • Use proportions with a triangle or parallel lines.

Your Notes

THEOREM 6.4: TRIANGLE PROPORTIONALITY THEOREM

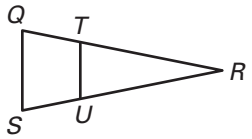
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

THEOREM 6.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

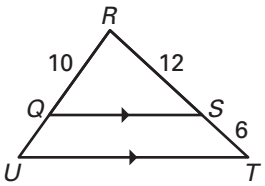
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Example 1 ***Find the length of a segment***

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RQ = 10$, $RS = 12$, and $ST = 6$. What is the length of $\overline{QU}$?



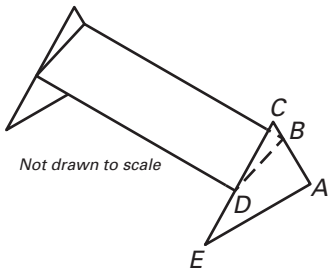
Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$	Triangle Proportionality Theorem
$\frac{10}{QU} = \frac{12}{6}$	**Substitute.**
$60 = 12 \cdot QU$	**Cross Products Property**
$5 = QU$	**Divide each side by** 12.

Your Notes

Example 2 ***Solve a real-world problem***

Aerodynamics A spoiler for a remote controlled car is shown where $AB = 31$ mm, $BC = 19$ mm, $CD = 27$ mm, and $DE = 23$ mm. *Explain* why $\overline{BD}$ is not parallel to $\overline{AE}$.



Solution

Find and simplify the ratios of lengths determined by $\overline{BD}$.

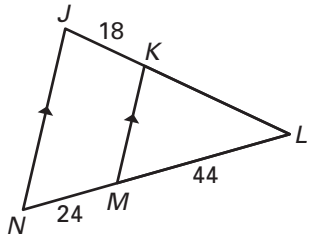
$\frac{CD}{DE} = \frac{27}{23}$ $\qquad$ $\frac{CB}{BA} = \frac{31}{19}$

Because $\frac{27}{23} \neq \frac{31}{19}$, $\overline{BD} \nparallel \overline{AE}$.

✔ ***Checkpoint*** **Complete the following exercises.**

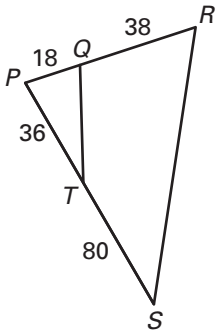
1. Find the length of $\overline{KL}$.

$KL = 33$



2. Determine whether $\overline{QT} \parallel \overline{RS}$.

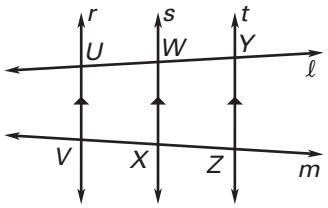
No; $\overline{QT}$ is not parallel to $\overline{RS}$.



Your Notes

THEOREM 6.6

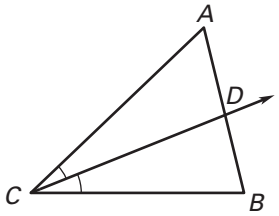
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

THEOREM 6.7

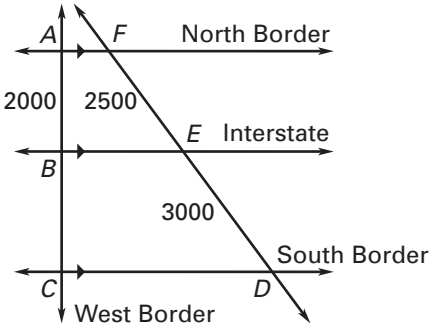
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$$\frac{AD}{DB} = \frac{CA}{CB}$$

Example 3 ***Use Theorem 6.6***

Farming A farmer's land is divided by a newly constructed interstate. The distances shown are in meters. Find the distance *CA* between the north border and the south border of the farmer's land.



Use Theorem 6.6.

$$\frac{CB}{BA} = \frac{DE}{EF}$$ **Parallel lines divide transversals proportionally.**

$$\frac{\boxed{CB + BA}}{BA} = \frac{\boxed{DE + EF}}{EF}$$ **Property of proportions (Property 4)**

$$\frac{CA}{\boxed{2000}} = \frac{3000 + 2500}{2500}$$ **Substitute.**

$$\frac{CA}{\boxed{2000}} = \frac{5500}{2500}$$ **Simplify.**

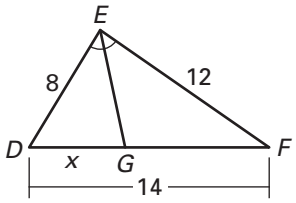
$$CA = 4400$$ **Multiply each side by 2000 and simplify.**

The distance between the north border and the south border is 4400 meters.

Your Notes

Example 4 *Use Theorem 6.7*

In the diagram, $\angle DEG \cong \angle GEF$. Use the given side lengths to find the length of $\overline{DG}$.



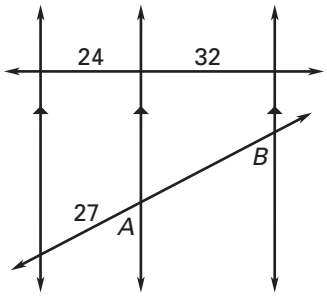
Solution

Because $\overrightarrow{EG}$ is an angle bisector of $\angle DEF$, you can apply Theorem 6.7. Let $GD = x$. Then $GF =$ $14 - x$.

$\frac{GF}{GD} = \frac{EF}{ED}$	**Angle bisector divides opposite side proportionally.**
$\frac{14 - x}{x} = \frac{12}{8}$	**Substitute.**
$12x = 112 - 8x$	**Cross Products Property**
$x = 5.6$	**Solve for x.**

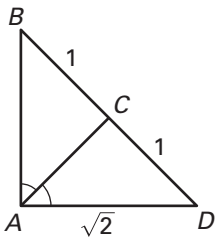
✔ ***Checkpoint*** **Find the length of $\overline{AB}$.**

3.



$AB = 36$

4.



$AB = \sqrt{2}$

Homework

6.7 Similarity Transformations and Coordinate Geometry

Goal • Perform dilations.

Your Notes

VOCABULARY

Dilation A dilation is a transformation that stretches or shrinks a figure to create a similar figure.

Center of dilation In a dilation, a figure is enlarged or reduced with respect to a fixed point called the center of dilation.

Scale factor of a dilation The scale factor k of a dilation is the ratio of a side length of the image to the corresponding side length of the original figure.

Reduction A dilation where $0 < k < 1$ is a reduction.

Enlargement A dilation where $k > 1$ is an enlargement.

COORDINATE NOTATION FOR A DILATION

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a reduction. If $k > 1$, the dilation is an enlargement.

Your Notes

All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Example 1 ***Draw a dilation with a scale factor greater than 1***

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 0), *B*(6, −4), *C*(8, 2), and *D*(6, 4). Use a scale factor of $\frac{1}{2}$.

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by $\frac{1}{2}$. Then draw the dilation.

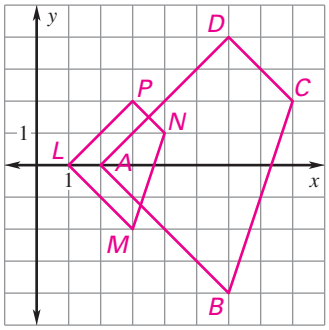
$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

A(2, 0) → *L* (1, 0)

B(6, −4) → *M* (3, −2)

C(8, 2) → *N* (4, 1)

D(6, 4) → *P* (3, 2)



Example 2 ***Verify that a figure is similar to its dilation***

A triangle has the vertices *A*(2, −1), *B*(4, −1), and *C*(4, 2). The image of △*ABC* after a dilation with a scale factor of 2 is △*DEF*.

a. Sketch △*ABC* and △*DEF*.

b. Verify that △*ABC* and △*DEF* are similar.

Solution

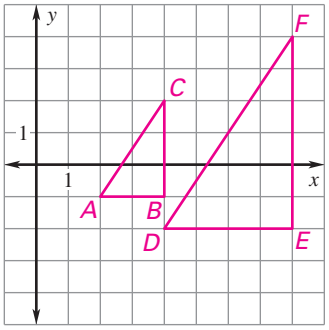
a. The scale factor is greater than 1, so the dilation is an enlargement.

$(x, y) \rightarrow (2x, 2y)$

A(2, −1) → *D*(4, −2)

B(4, −1) → *E*(8, −2)

C(4, 2) → *F*(8, 4)



b. Because ∠*B* and ∠*E* are both right angles, ∠*B* ≅ ∠*E*. Show that the lengths of the sides that include ∠*B* and ∠*E* are proportional.

$$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \qquad \frac{2}{4} = \frac{3}{6} \checkmark$$

The lengths are proportional. So, △*ABC* ~ △*DEF* by the SAS Similarity Theorem.

Your Notes

Example 3 ***Find a scale factor***

Magnets You are making your own photo magnets. Your photo is 8 inches by 10 inches. The image on the magnet is 2.8 inches by 3.5 inches. What is the scale factor of the reduction?

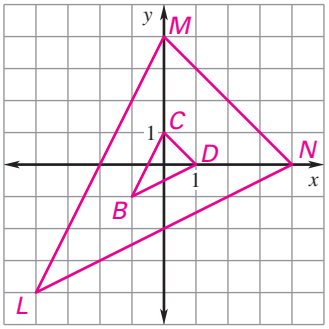
Solution

The scale factor is the ratio of a side length of the magnet image to a side length of the original photo, or $\frac{2.8 \text{ in.}}{8 \text{ in.}}$. In simplest form, the scale factor is $\frac{7}{20}$.

✔ ***Checkpoint*** **Complete the following exercises.**

1. A triangle has the vertices $B(-1, -1)$, $C(0, 1)$, and $D(1, 0)$. Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle BCD$ with a scale factor of 4. Sketch $\triangle BCD$ and $\triangle LMN$.

$L(-4, -4)$,
$M(0, 4)$,
$N(4, 0)$



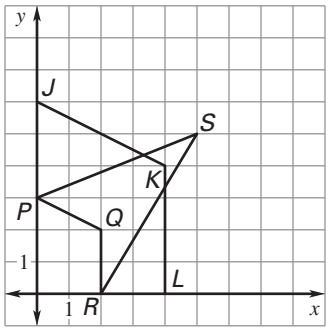
2. In Example 3, what is the scale factor of the reduction if your photo is 4 inches by 5 inches?

$\frac{7}{10}$

Your Notes

Example 4 ***Find missing coordinates***

You want to create a quadrilateral *JKLM* that is similar to quadrilateral *PQRS*. What are the coordinates of *M*?



Solution

Determine if *JKLM* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *J*, *K*, and *L* from *P*, *Q*, and *R*.

$(x, y) \rightarrow (kx, ky)$

P(0, 3) → *J*(0, 6) $k =$ 2

Q(2, 2) → *K*(4, 4) $k =$ 2

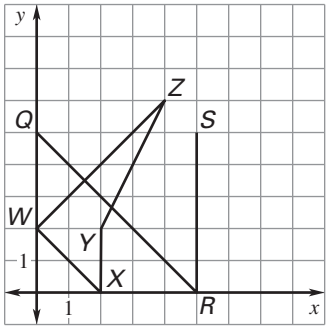
R(2, 0) → *L*(4, 0) $k =$ 2

Because *k* is the same in each case, the image is a dilation with a scale factor of 2. So, you can use the scale factor to find the image *M* of point *S*.

S(5, 5) → *M*(2 • 5 , 2 • 5) = *M*(10, 10)

✔ ***Checkpoint*** **Complete the following exercise.**

3. You want to create a quadrilateral *QRST* that is similar to quadrilateral *WXYZ*. What are the coordinates of *T*?

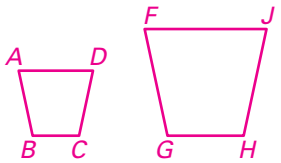
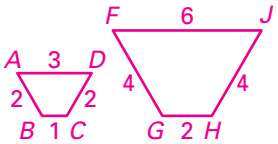


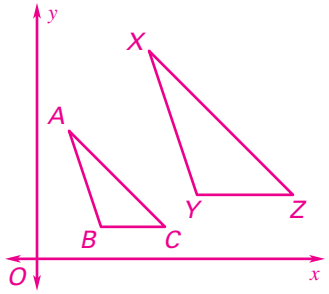
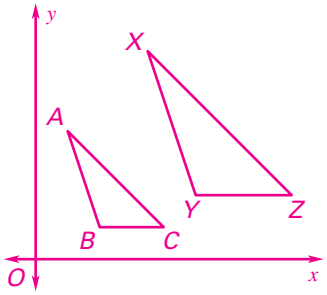
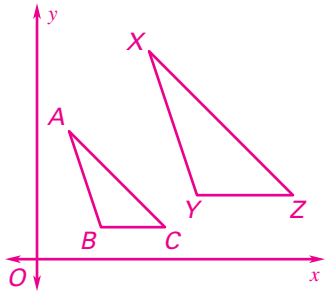
T(10, 15)

Homework

Words to Review

Give an example of the vocabulary word.

Ratio $\frac{4}{3}$ or 4 : 3	**Proportion** $\frac{x}{4} = \frac{3}{12}$
Means The means of $\frac{a}{b} = \frac{c}{d}$ are b and c.	**Extremes** The extremes of $\frac{a}{b} = \frac{c}{d}$ are a and d.
Geometric mean The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.	**Scale drawings** A scale drawing is a drawing that is the same shape as the object it represents. A map is an example of a scale drawing.
Scale The scale of a map is 1 inch to 25 miles.	**Similar polygons**  $ABCD \sim FGHJ$
Scale factor of two similar polygons  The scale factor of $ABCD$ to $FGHJ$ is $\frac{1}{2}$.	**Fibonacci sequence** 1,1,2,3,5,8,13,...
Golden ratio 1.618...	

Golden rectangle	Dilation
	
Center of dilation	**Scale factor of a dilation**
 The center of dilation is (0, 0).	 The scale factor of the dilation is $\frac{XY}{AB}$.
Reduction	**Enlargement**
A dilation with a scale factor greater than 0 and less than 1 is a reduction.	A dilation with a scale factor greater than 1 is an enlargement.

Review your notes and Chapter 6 by using the Chapter Review on pages 435–438 of your textbook.