Goal • Name and sketch geometric figures.

VOCABULARY

Undefined term A word without a formal definition

Point A point has no dimension. It is represented by a dot.

Line A line has one dimension. It is represented by a line with two arrowheads.

Plane A plane has two dimensions. It is represented by a shape that looks like a floor or a wall.

Collinear points Points that lie on the same line

Coplanar points Points that lie in the same plane

Defined Terms Terms that can be described using known words

Line segment, endpoints Part of a line that consists of two points, called endpoints, and all the points on the line between the endpoints

Ray The ray $AB$ consists of the endpoint $A$ and all points on $\overline{AB}$ that lie on the same side of $A$ as $B$.

Opposite rays If point $C$ lies on $\overline{AB}$ between $A$ and $B$, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

Intersection The intersection of two or more geometric figures is the set of points that the figures have in common.
UNDEFINED TERMS

Point  A point has **no** dimension. It is represented by a **dot**.

Line  A line has **one** dimension. It is represented by a **line** with two arrowheads, but it extends without end.

Through any **two** points, there is exactly **one** line. You can use any **two** points on a line to name it.

Plane  A plane has **two** dimensions. It is represented by a shape that looks like a floor or wall, but it extends without end.

Through any **three** points not on the same line, there is exactly **one** plane. You can use **three** points that are not all on the same line to name a plane.

**Example 1**  **Name points, lines, and planes**

a. Give two other names for \( \overrightarrow{LN} \). Give another name for plane \( Z \).

b. Name three points that are collinear. Name four points that are coplanar.

a. Other names for \( \overrightarrow{LN} \) are \( \overrightarrow{LM} \) and **line b**. Other names for plane \( Z \) are **plane LMP** and **LNP**.

b. Points \( L, M, \) and \( N \) lie on the same line, so they are collinear. Points \( L, M, N, \) and \( P \) lie on the same plane, so they are coplanar.

**Checkpoint**  Use the diagram in Example 1.

1. Give two other names for \( \overrightarrow{MQ} \). Name a point that is not coplanar with points \( L, N, \) and \( P \).

   \( QM \) and line \( a; \) point \( Q \)
**DEFINED TERMS: SEGMENTS AND RAYS**

Line \( AB \) (written as \( \overline{AB} \)) and points \( A \) and \( B \) are used here to define the terms below.

**Segment** The line segment \( AB \), or segment \( AB \), (written as \( \overline{AB} \)) consists of the endpoints \( A \) and \( B \) and all points on \( \overrightarrow{AB} \) that are between \( A \) and \( B \).

Note that \( AB \) can also be named \( BA \).

**Ray** The ray \( AB \) (written as \( \overrightarrow{AB} \)) consists of the endpoint \( A \) and all points on \( \overrightarrow{AB} \) that lie on the same side of \( A \) as \( B \).

Note that \( AB \) and \( BA \) are different rays.

---

**Example 2** Name segments, rays, and opposite rays

a. Give another name for \( VX \).

b. Name all rays with endpoints \( W \).

Which of these rays are opposite rays?

a. Another name for \( VX \) is \( XV \).

b. The rays with endpoint \( W \) are \( WV, WY, WX, \) and \( WZ \). The opposite rays with endpoint \( W \) are \( WV \) and \( WX \), and \( WY \) and \( WZ \).

**Checkpoint** Use the diagram in Example 2.

2. Give another name for \(YW\).

\( WY \)

3. Are \( VX \) and \( XV \) the same ray? Are \( VW \) and \( VX \) the same ray? Explain.

No, the rays do not have the same endpoint; Yes, the rays have a common endpoint, are collinear, and consist of the same points.
**Example 3 Sketch intersections of lines and planes**

a. Sketch a plane and a line that intersects the plane at more than one point.

b. Sketch a plane and a line that is in the plane. Sketch another line that intersects the line and plane at a point.

![Sketch of a plane and a line intersecting at more than one point.]

![Sketch of a plane, a line in the plane, and another line intersecting them.]

**Example 4 Sketch intersections of planes**

Sketch two planes that intersect in a line.

**Step 1 Sketch** one plane as if you are facing it.

**Step 2 Sketch** a second plane that is horizontal. Use dashed lines to show where one plane is hidden.

**Step 3 Sketch** the line of intersection.

**Checkpoint** Complete the following exercises.

4. Sketch two different lines that intersect a plane at different points.

![Sketch of two lines intersecting a plane.]

5. Name the intersection of $\overline{MX}$ and line $a$.

   point $M$

6. Name the intersection of plane $C$ and plane $D$.

   line $a$
1.2 Use Segments and Congruence

**Goal**
- Use segment postulates to identify congruent segments.

### VOCABULARY

**Postulate, axiom** A rule that is accepted without proof

**Theorem** A rule that can be proved

**Coordinate** The real number that corresponds to a point

**Distance** The distance between two points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

**Between** When three points are collinear, you can say that one point is between the other two.

**Congruent segments** Line segments that have the same length

### POSTULATE 1  **RULER POSTULATE**

The points on a line can be matched one to one with real numbers. The real number that corresponds to a point is the coordinate of the point.

The distance between points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

$$AB = |x_2 - x_1|$$
Example 1  **Apply the Ruler Postulate**

Measure the length of CD to the nearest tenth of a centimeter.

**Solution**
Align one mark of a metric ruler with C. Then estimate the coordinate of D. For example, if you align C with 1, D appears to align with 4.7.

\[ CD = |4.7 - 1| = 3.7 \]

Ruler postulate

The length of CD is about 3.7 centimeters.

**POSTULATE 2  SEGMENT ADDITION POSTULATE**

If B is between A and C, then
\[ AB + BC = AC. \]
If \( AB + BC = AC \), then B is between A and C.

Example 2  **Apply the Segment Addition Postulate**

Road Trip  The locations shown lie in a straight line. Find the distance from the starting point to the destination.

**Solution**
The rest area lies between the starting point and the destination, so you can apply the Segment Addition Postulate.

\[ SD = SR + RD \]
\[ = 64 + 87 \]
\[ = 151 \]

Add.

The distance from the starting point to the destination is 151 miles.
Example 3  **Find a length**

Use the diagram to find $KL$.

Use the Segment Addition Postulate to write an equation. Then solve the equation to find $KL$.

$$JL = JK + KL \quad \text{Segment Addition Postulate}$$

$$38 = 15 + KL \quad \text{Substitute for } JL \text{ and } JK.$$  

$$23 = KL \quad \text{Subtract 15 from each side.}$$

Example 4  **Compare segments for congruence**

Plot $F(4, 5)$, $G(-1, 5)$, $H(3, 3)$, and $J(3, -2)$ in a coordinate plane. Then determine whether $FG$ and $HJ$ are congruent.

Horizontal segment: Subtract the $x$-coordinates of the endpoints.

$$FG = |4 - (-1)| = 5$$

Vertical segment: Subtract the $y$-coordinates of the endpoints.

$$HJ = |3 - (-2)| = 5$$

$FG$ and $HJ$ have the same length. So $FG \cong HJ$.

**Checkpoint** Complete the following exercises.

1. Find the length of $AB$ to the nearest $\frac{1}{8}$ inch. Then draw a segment with the same length.

   $1\frac{7}{8}$ inches; Check drawing

2. Find $QS$ and $PQ$.

   $61; 24$

3. Consider the points $A(-2, -1)$, $B(4, -1)$, $C(3, 0)$, and $D(3, 5)$. Are $AB$ and $CD$ congruent? No
1.3 Use Midpoint and Distance Formulas

Goal • Find lengths of segments in the coordinate plane.

VOCABULARY

Midpoint The point that divides a segment into two congruent segments

Segment bisector A point, ray, line, line segment, or plane that intersects the segment at its midpoint

Example 1 Find segments lengths

Find \( RS \).

\begin{align*}
\text{Solution} & \quad T \text{ is the midpoint of } RS. \text{ So, } RT = TS = 21.7. \\
RS & = RT + TS \quad \text{Segment Addition Postulate} \\
& = 21.7 + 21.7 \quad \text{Substitute.} \\
& = 43.4 \quad \text{Add.}
\end{align*}

The length of \( RS \) is 43.4.

Checkpoint Complete the following exercise.

1. Find \( AB \).
Point C is the midpoint of $BD$. Find the length of $BC$.

**Solution**

Step 1 Write and solve an equation.

$$BC = CD$$

Write equation.

$$\frac{3x - 2}{x - 2} = \frac{2x + 3}{3}$$

Substitute.

$$x = \frac{5}{2}$$

Add 2 to each side.

Step 2 Evaluate the expression for $BC$ when $x = 5$.

$$BC = \frac{3x - 2}{x - 2} = \frac{3(5) - 2}{5 - 2} = 13$$

So, the length of $BC$ is 13 units.

**Checkpoint** Complete the following exercise.

2. Point $K$ is the midpoint of $JL$. Find the length of $KL$.

$$\frac{8 - 3x}{2x + 5} = \frac{6}{5}$$

**THE MIDPOINT FORMULA**

The coordinates of the midpoint of a segment are the averages of the $x$-coordinates and of the $y$-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint $M$ of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$
Example 3  Use the Midpoint Formula

a. Find Midpoint  The endpoints of $PR$ are $P(−2, 5)$ and $R(4, 3)$. Find the coordinates of the midpoint $M$.

b. Find Endpoint  The midpoint of $AC$ is $M(3, 4)$. One endpoint is $A(1, 6)$. Find the coordinates of endpoint $C$.

Solution

a. Use the Midpoint Formula.

\[
M = \left( \frac{-2 + 4}{2}, \frac{5 + 3}{2} \right) = M(1, 4)
\]

The coordinates of the midpoint of $PR$ are $M(1, 4)$.

b. Let $(x, y)$ be the coordinates of endpoint $C$. Use the Midpoint Formula to find $x$ and $y$.

Step 1  Find $x$.  
Step 2  Find $y$.

\[
\frac{1 + x}{2} = \frac{3}{2}  \quad \frac{6 + y}{2} = \frac{4}{2}
\]

\[
1 + x = 6  \quad 6 + y = 8
\]

\[
x = 5  \quad y = 2
\]

The coordinates of endpoint $C$ are $(5, 2)$.

Checkpoint  Complete the following exercises.

3. The endpoints of $CD$ are $C(−8, −1)$ and $D(2, 4)$. Find the coordinates of the midpoint $M$.

\[
M(-3, \frac{3}{2})
\]

4. The midpoint of $XZ$ is $M(5, −6)$. One endpoint is $X(−3, 7)$. Find the coordinates of endpoint $Z$.

\[
(13, −19)
\]
THE DISTANCE FORMULA

If \( A(x_1, y_1) \) and \( B(x_2, y_2) \) are points in a coordinate plane, then the distance between \( A \) and \( B \) is

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Example 4  Use the Distance Formula

What is the approximate length of \( RT \), with endpoints \( R(3, 2) \) and \( T(-4, 3) \)?

Solution

Use the Distance Formula.

\[
RT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(-4 - 3)^2 + (3 - 2)^2}
\]

\[
= \sqrt{(-7)^2 + (1)^2}
\]

\[
= \sqrt{49 + 1}
\]

\[
= \sqrt{50}
\]

\[
\approx 7.07
\]

The length of \( RT \) is about 7.07.

Checkpoint  Complete the following exercise.

5. What is the approximate length of \( GH \), with endpoints \( G(5, -1) \) and \( H(-3, 6) \)?

about 10.63
Lesson 1.4 • Measure and Classify Angles

Goal
• Name, measure, and classify angles.

Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle</td>
<td>An angle consists of two different rays with the same endpoint.</td>
</tr>
<tr>
<td>Sides of an angle</td>
<td>In an angle, the rays are called the sides of the angle.</td>
</tr>
<tr>
<td>Vertex of an angle</td>
<td>In an angle, the endpoint is the vertex of the angle.</td>
</tr>
<tr>
<td>Measure of an angle</td>
<td>In ∠AOB, OA and OB can be matched one to one with real numbers from 0 to 180. The measure of ∠AOB is equal to the absolute value of the difference between the real numbers for OA and OB.</td>
</tr>
<tr>
<td>Acute angle</td>
<td>An angle that measures between 0° and 90°</td>
</tr>
<tr>
<td>Right angle</td>
<td>An angle that measures 90°</td>
</tr>
<tr>
<td>Obtuse angle</td>
<td>An angle that measures between 90° and 180°</td>
</tr>
<tr>
<td>Straight angle</td>
<td>An angle that measures 180°</td>
</tr>
<tr>
<td>Congruent angles</td>
<td>Angles with the same measure</td>
</tr>
<tr>
<td>Angle bisector</td>
<td>A ray that divides an angle into two angles that are congruent</td>
</tr>
</tbody>
</table>

Example 1
Name angles

Name the three angles in the diagram.

∠ABC, or ∠CBA
∠CBD, or ∠DBC
∠ABD, or ∠DBA
POSTULATE 3: PROTRACTOR POSTULATE

Consider \( \overrightarrow{OB} \) and point \( A \) on one side of \( \overrightarrow{OB} \). The rays of the form \( \overrightarrow{OA} \) can be matched one to one with the real numbers from 0 to 180. The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \).

Example 2  Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. \( \angle WSR \)  
   b. \( \angle TSW \)

c. \( \angle RST \)  
   d. \( \angle VST \)

a. \( \overrightarrow{SR} \) is lined up with the 0° on the outer scale of the protractor. \( \overrightarrow{SW} \) passes through 65° on the outer scale. So, \( m\angle WSR = 65^\circ \). It is an acute angle.

b. \( \overrightarrow{ST} \) is lined up with the 0° on the inner scale of the protractor. \( \overrightarrow{SW} \) passes through 115° on the inner scale. So, \( m\angle TSW = 115^\circ \). It is an obtuse angle.

c. \( m\angle RST = 180^\circ \). It is a straight angle.

d. \( m\angle VST = 90^\circ \). It is a right angle.

Checkpoint  Complete the following exercises.

1. Name all the angles in the diagram at the right.
   \[ \angle FGH \text{ or } \angle HGF, \angle FGJ \text{ or } \angle JGF, \angle JGH \text{ or } \angle HGJ \]

2. What type of angles do the \( x \)-axis and \( y \)-axis form in a coordinate plane?
   right angles
**POSTULATE 4: ANGLE ADDITION POSTULATE**

Words: If \( P \) is in the interior of \( \angle RST \), then the measure of \( \angle RST \) is equal to the sum of the measures of \( \angle RSP \) and \( \angle PST \).

Symbols: If \( P \) is in the interior of \( \angle RST \), then \( \measuredangle RST = \measuredangle RSP + \measuredangle PST \).

---

**Example 3** Find angle measures

Given that \( \measuredangle GFJ = 155^\circ \), find \( \measuredangle GFH \) and \( \measuredangle HFJ \).

**Solution**

Step 1 Write and solve an equation to find the value of \( x \).

\[
\measuredangle GFJ = \measuredangle GFH + \measuredangle HFJ \quad \text{Angle Addition Postulate} \\
155^\circ = (4x + 4)^\circ + (4x - 1)^\circ \\
155 = 8x + 3 \\
152 = 8x \\
19 = x
\]

Step 2 Evaluate the given expressions when \( x = 19 \).

\[
\measuredangle GFH = (4x + 4)^\circ = (4 \cdot 19 + 4)^\circ = 80^\circ \\
\measuredangle HFJ = (4x - 1)^\circ = (4 \cdot 19 - 1)^\circ = 75^\circ \\
So, \measuredangle GFH = 80^\circ \text{ and } \measuredangle HFJ = 75^\circ.
\]

**Checkpoint** Complete the following exercise.

3. Given that \( \angle VRS \) is a right angle, find \( \measuredangle VRT \) and \( \measuredangle TRS \).

\[ \measuredangle VRT = 19^\circ, \measuredangle TRS = 71^\circ \]
Example 4  Identify congruent angles

Identify all pairs of congruent angles in the diagram. If $m\angle P = 120^\circ$, what is $m\angle N$?

Solution
There are two pairs of congruent angles:

$\angle P \cong \angle N$ and $\angle L \cong \angle M$

Because $\angle P \cong \angle N$, $m\angle P = m\angle N$.
So, $m\angle N = 120^\circ$.

Example 5  Double an angle measure

In the diagram at the right, $\overline{WY}$ bisects $\angle XWZ$, and $m\angle XWY = 29^\circ$.
Find $m\angle XWZ$.

Solution
By the Angle Addition Postulate,
$m\angle XWZ = m\angle XWY + m\angle YWZ$.

Because $\overline{WY}$ bisects $\angle XWZ$, you know $\angle XWY \cong \angle YWZ$.

So, $m\angle XWY = m\angle YWZ$, and you can write
$m\angle XWZ = m\angle XWY + m\angle YWZ$

$= 29^\circ + 29^\circ = 58^\circ$.

Checkpoint  Complete the following exercises.

4. Identify all pairs of congruent angles in the diagram. If $m\angle B = 135^\circ$, what is $m\angle D$?

$\angle B \cong \angle D$ and $\angle A \cong \angle C$, $135^\circ$

5. In the diagram below, $\overline{KM}$ bisects $\angle LKN$ and $m\angle LKM = 78^\circ$. Find $m\angle LKN$.

$156^\circ$
Describe Angle Pair Relationships

**Goal**  • Use special angle relationships to find angle measures.

**VOCABULARY**

<table>
<thead>
<tr>
<th>Complementary angles</th>
<th>Two angles whose sum is 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplementary angles</td>
<td>Two angles whose sum is 180°</td>
</tr>
<tr>
<td>Adjacent angles</td>
<td>Two angles that share a common vertex or side, but have no common interior points</td>
</tr>
<tr>
<td>Linear pair</td>
<td>Two adjacent angles are a linear pair if their noncommon sides are opposite rays.</td>
</tr>
<tr>
<td>Vertical angles</td>
<td>Two angles are vertical angles if their sides form two pairs of opposite rays.</td>
</tr>
</tbody>
</table>

**Example 1**  Identify complements and supplements

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

**Solution**

Because $52° + 38° = 90°$, $\angle ABD$ and $\angle CDB$ are complementary angles.

Because $52° + 128° = 180°$, $\angle ABD$ and $\angle EDB$ are supplementary angles.

Because $\angle CDB$ and $\angle BDE$ share a common vertex and side, they are adjacent angles.
Example 2  Find measures of complements and supplements

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 57^\circ$, find $m\angle 1$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 41^\circ$, find $m\angle 3$.

Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 1 = 90^\circ - m\angle 2 = 90^\circ - 57^\circ = 33^\circ$$

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 41^\circ = 139^\circ$$

**Checkpoint**  Complete the following exercises.

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

   complementary: $\angle DEF$ and $\angle ABC$; supplementary: $\angle FEG$ and $\angle ABC$; adjacent: $\angle DEF$ and $\angle FEG$

2. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 73^\circ$, find $m\angle 2$.

   $17^\circ$

3. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 37^\circ$, find $m\angle 3$.

   $143^\circ$
Basketball The basketball pole forms a pair of supplementary angles with the ground. Find $\angle BCA$ and $\angle DCA$.

Solution

Step 1 Use the fact that $180^\circ$ is the sum of the measures of supplementary angles.

$$m\angle BCA + m\angle DCA = 180^\circ$$

Write equation.

$$\begin{align*}
(3x + 8)^\circ + (4x - 3)^\circ &= 180^\circ \\
7x + 5 &= 180 \\
7x &= 175 \\
x &= 25
\end{align*}$$

Substitute.

Combine like terms.

Subtract.

Step 2 Evaluate the original expressions when $x = 25$.

$$m\angle BCA = (3x + 8)^\circ = (3 \cdot 25 + 8)^\circ = 83^\circ.$$  
$$m\angle DCA = (4x - 3)^\circ = (4 \cdot 25 - 3)^\circ = 97^\circ.$$  

The angle measures are $83^\circ$ and $97^\circ$.

Checkpoint Complete the following exercise.

4. In Example 3, suppose the angle measures are $(5x + 1)^\circ$ and $(6x + 3)^\circ$. Find $m\angle BCA$ and $m\angle DCA$.

$81^\circ$ and $99^\circ$
Example 4  Identify angle pairs

Identify all of the linear pairs and all of the vertical angles in the figure at the right.

Solution
To find vertical angles, look for angles formed by intersecting lines. 
\( \angle 1 \) and \( \angle 3 \) are vertical angles.
To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays. 
\( \angle 1 \) and \( \angle 2 \) are a linear pair. \( \angle 2 \) and \( \angle 3 \) are a linear pair.

Checkpoint  Complete the following exercise.

5. Identify all of the linear pairs and all of the vertical angles in the figure.

linear pairs: none; vertical angles: \( \angle 1 \) and \( \angle 4 \), \( \angle 2 \) and \( \angle 5 \), \( \angle 3 \) and \( \angle 6 \)

Example 5  Find angle measures in a linear pair

Two angles form a linear pair. The measure of one angle is 4 times the measure of the other. Find the measure of each angle.

Solution
Let \( x^\circ \) be the measure of one angle. The measure of the other angle is \( 4x^\circ \). Then use the fact that the angles of a linear pair are supplementary to write an equation.
\[
x^\circ + 4x^\circ = 180^\circ
\]
Write an equation.
\[
5x = 180
\]
Combine like terms.
\[
x = 36
\]
Divide each side by 5.
The measures of the angles are \( 36^\circ \) and \( 4(36^\circ) = 144^\circ \).
CONCEPT SUMMARY: INTERPRETING A DIAGRAM

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right.

- All points shown are **coplanar**.
- Points $A$, $B$, and $C$ are **collinear**, and $B$ is between $A$ and $C$.
- $\overrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ **intersect** at point $B$.
- $\angle DBE$ and $\angle EBC$ are **adjacent** angles, and $\angle ABC$ is a **straight angle**.
- Point $E$ lies in the **interior** of $\angle DBC$.

In the diagram above, you cannot conclude that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the measure of each angle.

$45^\circ$ and $135^\circ$
1.6 Classify Polygons

Goal • Classify polygons.

**VOCABULARY**

**Polygon**  A polygon is a closed plane figure with the following properties: (1) It is formed by three or more line segments called sides. (2) Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

**Sides**  The sides of a polygon are the line segments that form the polygon.

**Vertex**  A vertex of a polygon is an endpoint of a side of the polygon.

**Convex**  A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

**Concave**  A concave polygon is a polygon that is not convex.

**n-gon**  An n-gon is a polygon with n sides.

**Equilateral**  A polygon is equilateral if all of its sides are congruent.

**Equiangular**  A polygon is equiangular if all of its angles in the interior are congruent.

**Regular**  A polygon is regular if all sides and all angles are congruent.
IDENTIFYING POLYGONS

In geometry, a figure that lies in a plane is called a **plane figure**. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly **two** sides, one at each endpoint, so that no two sides with a common endpoint are **collinear**.

Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is **vertices**. A polygon can be named by listing the vertices in consecutive order. For example, $ABCDE$ and $CDEAB$ are both correct names for the polygon at the right.

**Example 1**  
*Identify polygons*

Tell whether the figure is a polygon and whether it is **convex** or **concave**.

a. ![Figure](image)

Solution

a. Some segments intersect more than two segments, so it is **not a polygon**.

b. The figure is **a convex polygon**.

c. The figure is **a concave polygon**.

**Checkpoint**  
Tell whether the figure is a polygon and whether it is **convex** or **concave**.

1. **convex polygon**

2. **not a polygon**
**Example 2**  **Classify polygons**

Classify the polygon by the number of sides. Tell which terms apply to the polygon: *equilateral, equiangular, regular*, or *not regular*.

**Solution**

The polygon has **8** sides. It is equilateral and equiangular, so it is a **regular octagon**.

---

**Example 3**  **Find side lengths**

The head of a bolt is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal bolt. Find the length of a side.

**Solution**

First, write and solve an equation to find the value of $x$. Use the fact that the sides of a regular hexagon are **congruent**.

$$4x + 3 = 5x - 1$$  
Write an equation.

$$4 = x$$  
Simplify.

Then evaluate one of the expressions to find a side length when $x = 4$.  

$$4x + 3 = 4(4) + 3 = 19$$  
The length of a side is **19** millimeters.

---

**Checkpoint**  Complete the following exercises.

3. Classify the polygon by the number of sides. Tell which terms apply to the polygon: *equilateral, equiangular, regular*, or *not regular*.

   quadrilateral; not regular

4. The expressions $(4x + 8)°$ and $(5x - 5)°$ represent the measures of two of the congruent angles in Example 3. Find the measure of an angle.

   $60°$
1.7 Find Perimeter, Circumference, and Area

**Goal** • Find dimensions of polygons.

### Your Notes

**FORMULAS FOR PERIMETER P, AREA A, AND CIRCUMFERENCE C**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>( P = 4s )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A = s^2 )</td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td>( P = 2l + 2w )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A = lw )</td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td>( P = a + b + c )</td>
<td>( P = 2(78) + 2(27) = 210 )</td>
</tr>
<tr>
<td></td>
<td>( A = \frac{1}{2}bh )</td>
<td>( A = \frac{1}{2} \times 78 \times 27 = 2106 )</td>
</tr>
<tr>
<td>Circle</td>
<td>( C = 2\pi r )</td>
<td>( C = 2\pi \times 78 )</td>
</tr>
<tr>
<td></td>
<td>( A = \pi r^2 )</td>
<td>( A = \pi \times 78^2 )</td>
</tr>
</tbody>
</table>

**Example 1**

Find the perimeter and area of a rectangle

Tennis The in-bounds portion of a singles tennis court is shown. Find its perimeter and area.

The in-bounds portion of a singles tennis court is shown. Find its perimeter and area.

**Perimeter**

\[ P = 2l + 2w \]

\[ P = 2(78) + 2(27) = 210 \]

**Area**

\[ A = lw \]

\[ A = 78 \times 27 = 2106 \]

The perimeter is **210** ft and the area is **2106** ft\(^2\).

**Checkpoint** Complete the following exercise.

1. In Example 1, the width of the in-bounds rectangle increases to 36 feet for doubles play. Find the perimeter and area of the in-bounds rectangle.

perimeter: 228 ft, area: 2808 ft\(^2\)
Archery  The smallest circle on an Olympic target is 12 centimeters in diameter. Find the approximate circumference and area of the smallest circle.

**Solution**
First find the radius. The diameter is 12 centimeters, so the radius is \( \frac{1}{2} (12) = 6 \) centimeters.

Then find the circumference and area. Use 3.14 for \( \pi \).

\[
P = 2\pi r \approx 2(3.14)(6) = 37.68 \text{ cm}
\]
\[
A = \pi r^2 \approx 3.14 (6)^2 = 113.04 \text{ cm}^2
\]

**Checkpoint** Find the approximate circumference and area of the circle.

2. \( C \approx 50.24 \text{ m}; A \approx 200.96 \text{ m}^2 \)

Triangle \( JKL \) has vertices \( J(1, 6), K(6, 6), \) and \( L(3, 2) \). Find the approximate perimeter of triangle \( JKL \).

**Solution**
First draw triangle \( JKL \) in a coordinate plane. Then find the side lengths. Because \( JK \) is horizontal, use the Ruler Postulate to find \( JK \). Use the Distance Formula to find \( JL \) and \( LK \).

\[
JK = \left| 6 - 1 \right| = 5 \text{ units}
\]
\[
JL = \sqrt{(3 - 1)^2 + (2 - 6)^2} = \sqrt{20} \approx 4.5 \text{ units}
\]
\[
LK = \sqrt{(6 - 3)^2 + (6 - 2)^2} = \sqrt{25} = 5 \text{ units}
\]

Then find the perimeter.

\[
P = JK + JL + LK \approx 5 + 4.5 + 5 = 14.5 \text{ units}.
\]
Lawn care You are using a roller to smooth a lawn. You can roll about 125 square yards in one minute. About how many minutes does it take to roll a lawn that is 120 feet long and 75 feet wide?

**Solution**
You can roll the lawn at a rate of 125 square yards per minute. So, the amount of time it takes you to roll the lawn depends on its area.

**Step 1** Find the area of the rectangular lawn.

\[
\text{Area} = \ell w = 120 \times 75 = 9000 \text{ ft}^2
\]

The rolling rate is in square yards per minute. Rewrite the area of the lawn in square yards. There are 3 feet in 1 yard, and \(3^2 = 9\) square feet in one square yard.

\[
9000 \text{ ft}^2 \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 1000 \text{ yd}^2 \quad \text{Use unit analysis.}
\]

**Step 2** Write a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let \(t\) represent the total time (in minutes) needed to roll the lawn.

\[
\begin{align*}
\text{Area of lawn (yd}^2\text{)} & = \text{Rolling rate (yd}^2\text{ per min)} \times \text{Total time (min)} \\
1000 & = 125 \cdot t \\
8 & = t
\end{align*}
\]

Substitute. Divide each side by 125.

It takes about 8 minutes to roll the lawn.
Example 5  Find unknown length

The base of a triangle is 24 feet. Its area is 216 square feet. Find the height of the triangle.

Solution

\[ A = \frac{1}{2}bh \]

Area of a triangle

\[
\begin{align*}
216 &= \frac{1}{2}(24)(h) \\
216 &= 12h \\
18 &= h
\end{align*}
\]

The height is 18 feet.

Checkpoint  Complete the following exercises.

3. Find the perimeter of the triangle shown at the right.

   about 17.1 units

4. Suppose a lawn is half as long and half as wide as the lawn in Example 4. Will it take half the time to roll the lawn? Explain.

   No, it will take a quarter of the time to roll the lawn because it is a quarter of the original area.

5. The area of a triangle is 96 square inches, and its height is 8 inches. Find the length of its base.

   24 inches
## Words to Review

**Give an example of the vocabulary word.**

<table>
<thead>
<tr>
<th>Point, line, plane</th>
<th>Collinear points</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>A and B are collinear points.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coplanar points</th>
<th>Line segment, endpoints</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td>D and T are coplanar points.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ray</th>
<th>Opposite rays</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
</tr>
<tr>
<td>XY is a ray with initial point X.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Postulate, axiom</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
</tr>
<tr>
<td>The intersection of two different lines is a point.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image9.png" alt="Diagram" /></td>
<td><img src="image10.png" alt="Diagram" /></td>
</tr>
<tr>
<td>The coordinates of points A and B are (x_1) and (x_2).</td>
<td></td>
</tr>
</tbody>
</table>

\[
AB = |x_2 - x_1| 
\]

The distance between points A and B is \(|x_2 - x_1|\).
<table>
<thead>
<tr>
<th>Between</th>
<th>Congruent segments</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of points A, C, and B" /></td>
<td><img src="image" alt="Diagram of segments AB and CD" /></td>
</tr>
<tr>
<td>Point C is between Points A and B.</td>
<td>AB and CD are congruent.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Midpoint</th>
<th>Segment bisector</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of midpoint M" /></td>
<td><img src="image" alt="Diagram of segment bisector FG" /></td>
</tr>
<tr>
<td>M is the midpoint of AB.</td>
<td>FG is a segment bisector of AB.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle, sides, vertex</th>
<th>Measure of an angle</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of sides AB and AC" /></td>
<td><img src="image" alt="Diagram of angle A" /></td>
</tr>
<tr>
<td>Sides AB and AC form ( \angle A ). The vertex is A.</td>
<td>The measure of ( \angle A ) is 40°.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acute angle</th>
<th>Right angle</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of acute angle" /></td>
<td><img src="image" alt="Diagram of right angle" /></td>
</tr>
<tr>
<td>( 0° &lt; m\angle A &lt; 90° )</td>
<td>( m\angle A = 90° )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Obtuse angle</th>
<th>Straight angle</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of obtuse angle" /></td>
<td><img src="image" alt="Diagram of straight angle" /></td>
</tr>
<tr>
<td>( 90° &lt; m\angle A &lt; 180° )</td>
<td>( m\angle A = 180° )</td>
</tr>
<tr>
<td>Angle bisector, congruent angles</td>
<td>Supplementary angles, linear pair</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>$BD$ is an angle bisector of $\angle CBE$.</td>
<td>$\angle STX$ and $\angle XTY$ are supplementary.</td>
</tr>
<tr>
<td>$\angle CBD$ and $\angle DBE$ are congruent.</td>
<td>$\angle STX$ and $\angle XTY$ are a linear pair.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Complementary angles, adjacent angles</th>
<th>Vertical angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\angle QPR$ and $\angle RPS$ are complementary.</td>
<td>$\angle 1$ and $\angle 2$ are vertical angles.</td>
</tr>
<tr>
<td>$\angle QPR$ and $\angle RPS$ are adjacent.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Polygon, side, vertex</th>
<th>Concave, convex</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ABCD$ is concave.</td>
<td>$FGHJ$ is convex.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$n$-gon</th>
<th>Equilateral, equiangular, regular</th>
</tr>
</thead>
<tbody>
<tr>
<td>An $n$-gon is a polygon with $n$ sides.</td>
<td>The polygon is equilateral and equiangular, so it is regular.</td>
</tr>
</tbody>
</table>

Review your notes and Chapter 1 by using the Chapter Review on pages 61–64 of your textbook.
2.1 Use Inductive Reasoning

Goal • Describe patterns and use inductive reasoning.

VOCABULARY

Conjecture  A conjecture is an unproven statement that is based on observations.

Inductive Reasoning  Inductive reasoning is the process of finding a pattern for specific cases and then writing a conjecture for the general case.

Counterexample  A counterexample is a specific case for which the conjecture is false.

Example 1  Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1  Figure 2  Figure 3

Solution

Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eightths. Shade the section just below the horizontal segment at the left.

Checkpoint  Complete the following exercise.

1. Sketch the fifth figure in the pattern in Example 1.

Figure 5
Example 2  
Describe the number pattern

Describe the pattern in the numbers $-1, -4, -16, -64, \ldots$. Write the next three numbers in the pattern.

Notice that each number in the pattern is four times the previous number.

$-1, -4, -16, -64, \ldots$

$\times 4 \quad \times 4 \quad \times 4 \quad \times 4$

The next three numbers are $-256, -1024, \text{ and } -4096$.

Example 3  
Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture</td>
<td>.</td>
<td>±</td>
<td>▽</td>
<td>✫</td>
<td>✫</td>
</tr>
<tr>
<td>Number of connections</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>?</td>
</tr>
</tbody>
</table>

$+1 \quad +2 \quad +3 \quad +?$

Conjecture: You can connect five noncollinear points $6 + 4$, or $10$ different ways.

✓ Checkpoint  Complete the following exercises.

2. Describe the pattern in the numbers $1, 2.5, 4, 5.5, \ldots$ and write the next three numbers in the pattern.

The numbers are increasing by $1.5; 7, 8.5, 10$.

3. Rework Example 3 if you are given six noncollinear points.

15 different ways
Example 4  Make and test a conjecture

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 Find a pattern using groups of small numbers.

\[
\begin{array}{c|c}
1 + 3 + 5 &= 9 \\ &= 3 \cdot 3 \\
3 + 5 + 7 &= 15 \\ &= 5 \cdot 3 \\
5 + 7 + 9 &= 21 \\ &= 7 \cdot 3 \\
7 + 9 + 11 &= 27 \\ &= 9 \cdot 3 \\
\end{array}
\]

Conjecture The sum of any three consecutive odd numbers is three times the second number.

Step 2 Test your conjecture using other numbers.

\[
\begin{array}{c}
-1 + 1 + 3 &= 3 = 1 \cdot 3 \checkmark \\
103 + 105 + 107 &= 315 = 105 \cdot 3 \checkmark \\
\end{array}
\]

Example 5  Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student’s conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

To find a counterexample, you need to find a difference that is greater than the larger number.

\[
8 - (-4) = 12
\]

Because \(12 \not< 8\), a counterexample exists. The conjecture is false.
Example 6  Making conjectures from data displays

The scatter plot shows the average running speed of a runner over various time intervals. Make a conjecture based on the graph.

Solution

The scatter plot shows that the average speeds decrease as the time increases. So, one possible conjecture is that the average running speed over a 60-minute interval will be slower than over a 10-minute time interval.

Checkpoint  Complete the following exercises.

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

\[
\frac{4}{1} = 8
\]

6. Use the graph in Example 6 to make a conjecture that could be true. Give an explanation that supports your reasoning.

Sample answer: The average speed of the runner over a 60-minute interval will be less than 5 miles per hour; The average speed was less than 5 miles per hour over 50 minutes, and overall, the speeds decrease as the times increase.
## 2.2 Analyze Conditional Statements

**Goal**
- Write definitions as conditional statements.

### VOCABULARY

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional statement</td>
<td>A logical statement that has two parts, a hypothesis and a conclusion.</td>
</tr>
<tr>
<td>If-then form</td>
<td>A form of a conditional statement in which the “if” part contains the hypothesis and the “then” part contains the conclusion.</td>
</tr>
<tr>
<td>Hypothesis</td>
<td>A hypothesis is the “if” part of a conditional statement.</td>
</tr>
<tr>
<td>Conclusion</td>
<td>A conclusion is the “then” part of a conditional statement.</td>
</tr>
<tr>
<td>Negation</td>
<td>The negation of a statement is the opposite of the original statement.</td>
</tr>
<tr>
<td>Converse</td>
<td>The converse of a conditional statement is formed by switching the hypothesis and conclusion.</td>
</tr>
<tr>
<td>Inverse</td>
<td>The inverse of a conditional statement is formed by negating both the hypothesis and conclusion.</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>The contrapositive of a conditional statement is formed by writing the converse and then negating both the hypothesis and conclusion.</td>
</tr>
<tr>
<td>Equivalent statements</td>
<td>Equivalent statements are two statements that are both true or both false.</td>
</tr>
<tr>
<td>Perpendicular lines</td>
<td>Two lines that intersect to form a right angle are perpendicular lines.</td>
</tr>
<tr>
<td>Biconditional statement</td>
<td>A statement that contains the phrase “if and only if.”</td>
</tr>
</tbody>
</table>
Write an equivalent conditional statement in if-then form.
All vertebrates have a backbone.

**Solution**
First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.
If **an animal is a vertebrate**, then **it has a backbone**.

**Checkpoint** Write an equivalent conditional statement in if-then form.

1. All triangles have 3 sides.
   If a figure is a triangle, then it has 3 sides.

2. When \( x = 2, x^2 = 4 \).
   If \( x = 2 \), then \( x^2 = 4 \).

**Example 2** Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement “Olympians are athletes.” Decide whether each statement is true or false.

**Solution**
If-then form **If you are an Olympian, then you are an athlete.** True, Olympians are athletes.

Converse **If you are an athlete, then you are an Olympian.** False, not all athletes are Olympians.

Inverse **If you are not an Olympian, then you are not an athlete.** False, even if you are not an Olympian, you can still be an athlete.

Contrapositive **If you are not an athlete, then you are not an Olympian.** True, a person who is not an athlete cannot be an Olympian.
PERPENDICULAR LINES
Definition: If two lines intersect to form a right angle, then they are perpendicular lines.
The definition can also be written using the converse: If any two lines are perpendicular lines, then they intersect to form a right angle.
You can write “line \( l \) is perpendicular to line \( m \)” as \( l \perp m \).

Example 3  Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. \( AC \perp BD \)

b. \( \angle AED \) and \( \angle BEC \) are a linear pair.

Solution
a. The statement is true. The right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. The statement is false. Because \( \angle AED \) and \( \angle BEC \) are not adjacent angles, \( \angle AED \) and \( \angle BEC \) are not a linear pair.

Example 4  Write a biconditional

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution
Converse: If two lines are parallel, then they lie in the same plane and do not intersect.

Biconditional: Two lines are parallel if and only if they lie in the same plane and do not intersect.
3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement “Squares are rectangles.” Decide whether each statement is true or false.

If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.

Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.

Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.

Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.

4. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

   a. \( \angle GLK \) and \( \angle JLK \) are supplementary.
      (a) True; linear pairs of angles are supplementary.
      
   b. \( \overrightarrow{GJ} \perp \overrightarrow{HK} \)
      (b) False; it is not known that the lines intersect at right angles.

5. Write the statement below as an equivalent biconditional.

   Statement: If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.
   A student is in group A if and only if the student is a boy.
Apply Deductive Reasoning

Goal

• Form logical arguments using deductive reasoning.

Your Notes

VOCABULARY

Deductive Reasoning  Using facts, definitions, accepted properties, and the laws of logic to form a logical argument

LAWS OF LOGIC

Law of Detachment  If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis \( p \), then conclusion \( q \).  If these statements are true, then this statement is true.

If hypothesis \( q \), then conclusion \( r \).

If hypothesis \( p \), then conclusion \( r \).

Example 1  Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two angles have the same measure, then they are congruent. You know that \( m\angle A = m\angle B \).

b. Jesse goes to the gym every weekday. Today is Monday.

Solution

a. Because \( m\angle A = m\angle B \) satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, \( \angle A \cong \angle B \).

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is “If it is a weekday,” and the conclusion is “then Jesse goes to the gym.”

“Today is Monday” satisfies the hypothesis of the conditional statement, so you can conclude that Jesse will go to the gym today.
Example 2  Use the Law of Syllogism

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

b. If \(x^2 > 36\), then \(x^2 > 30\). If \(x > 6\), then \(x^2 > 36\).

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.

If Ron eats lunch today, then he will drink a glass of milk.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If \(x > 6\), then \(x^2 > 30\).

c. Neither statement’s conclusion is the same as the other statement’s hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

Checkpoint  Complete the following exercises.

1. If \(0^\circ < m\angle A < 90^\circ\), then \(A\) is acute. The measure of \(\angle A\) is \(38^\circ\). Using the Law of Detachment, what statement can you make?

\(\angle A\) is acute.

2. State the law of logic that is illustrated below.

If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

If you do your homework, then you can watch your favorite show.

Law of Syllogism
What conclusion can you make about the sum of an odd integer and an odd integer?

Solution

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

\[-3 + 5 = 2, -1 + 5 = 4, 3 + 5 = 8\]
\[-3 + (-5) = -8, 1 + (-5) = -4, 3 + (-5) = -2\]

Conjecture: Odd integer + Odd integer = _____ integer

Step 2 Let \(n\) and \(m\) each be any integer. Use deductive reasoning to show the conjecture is true.

\(2n\) and \(2m\) are _____ integers because any integer multiplied by 2 is _____.

\(2n - 1\) and \(2m + 1\) are _____ integers because \(2n\) and \(2m\) are _____ integers.

\((2n - 1) + (2m + 1)\) represents the sum of an _____ integer \(2n - 1\) and an _____ integer \(2m + 1\).

\((2n - 1) + (2m + 1) = 2(n + m)\)

The result is the product of _____ and an integer \(n + m\). So, \(2(n + m)\) is an _____ integer.

The sum of an odd integer and an odd integer is an _____ integer.

**Checkpoint** Complete the following exercise.

3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

The sum of a negative integer and itself is twice the integer; \(-n + (-n) = -2n = 2(-n)\).
Example 4  Compare inductive and deductive reasoning

Decide whether inductive or deductive reasoning is used to reach the conclusion. Explain your reasoning.

a. Each time Eric mows his lawn, it takes longer than 30 minutes. So the next time Eric mows his lawn, it will take longer than 30 minutes.

b. Birds are the only animals that have feathers. A penguin has feathers. So a penguin is a type of bird.

Solution

a. Inductive reasoning, because a pattern is used to reach the conclusion.

b. Deductive reasoning, because facts about animals and the laws of logic are used to reach a conclusion.

Checkpoint  Complete the following exercises.

4. Use inductive reasoning to write another statement about the graph in Example 4.
   
   Sample answer: The faster the average speed of the runner, the less time he or she is running.

5. Give an example of when you used deductive reasoning in everyday life.
   
   Check examples.
Symbolic Notation and Truth Tables

**Goal**
- Use symbolic notation to represent logical statements.

**VOCABULARY**

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truth value</td>
<td>Description of a statement as either true (T) or false (F)</td>
</tr>
<tr>
<td>Truth table</td>
<td>Table that shows the truth values of a conditional statement</td>
</tr>
<tr>
<td>Logically equivalent</td>
<td>Relationship describing two statements that have the same truth value</td>
</tr>
</tbody>
</table>

**SYMBOLIC NOTATION**

Let $p$ be “two angles are congruent” and let $q$ be “the measures of the angles are equal.”

**Conditional**
- If $p$, then $q$. $p \rightarrow q$
- If two angles are congruent, then their measures are equal.

**Converse**
- If $q$, then $p$. $q \rightarrow p$
- If the measures of two angles are equal, then they are congruent.

**Inverse**
- If not $p$, then not $q$. $\sim p \rightarrow \sim q$
- If two angles are not congruent, then their measures are not equal.

**Contrapositive**
- If not $q$, then not $p$. $\sim q \rightarrow \sim p$
- If the measures of two angles are not equal, then they are not congruent.

**Biconditional**
- $p$ if and only if $q$. $p \leftrightarrow q$
- Two angles are congruent if and only if their measures are equal.
Example 1  **Use symbolic notation**

Let \( p \) be “the sky is dark” and let \( q \) be “it is nighttime.”

a. Write the conditional statement \( p \rightarrow q \) in words.

b. Write the converse \( q \rightarrow p \) in words.

c. Write the biconditional \( p \rightarrow q \) in words.

**Solution**

a. If the sky is dark, then \( \text{it is nighttime} \).

b. If \( \text{it is nighttime} \), then \( \text{the sky is dark} \).

c. The sky is dark if and only if it is nighttime.

Example 2  **Make a truth table**

For a conditional statement with hypothesis \( p \) and conclusion \( q \), make a truth table for \( \sim q \rightarrow p \).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \sim q )</th>
<th>( \sim q \rightarrow p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

**Checkpoint** Complete the following exercises.

1. Write the inverse of the conditional statement, “If two angles are complementary, then their measures add up to 90°.”

If two angles are not complementary, then their measures do not add up to 90°.

2. For a conditional statement with hypothesis \( p \) and conclusion \( q \), make a truth table for \( \sim q \rightarrow \sim p \).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( \sim q )</th>
<th>( \sim p )</th>
<th>( \sim q \rightarrow \sim p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
2.4 Use Postulates and Diagrams

Goal • Use postulates involving points, lines, and planes.

Your Notes

**VOCABULARY**

Line perpendicular to a plane  A line is perpendicular to a plane if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

**POINT, LINE, AND PLANE POSTULATES**

Postulate 5  Through any two points there exists exactly one [line](#).

Postulate 6  A line contains at least two [points](#).

Postulate 7  If two lines intersect, then their intersection is exactly [one point](#).

Postulate 8  Through any three [noncollinear](#) points there exists exactly one plane.

Postulate 9  A plane contains at least three [noncollinear](#) points.

Postulate 10  If two points lie in a plane, then the line containing them [lies in the plane](#).

Postulate 11  If two planes intersect, then their intersection is a [line](#).

**Example 1** Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.

If \(\text{A} \bullet \text{B} \bullet \text{C} \bullet\) then \(\text{A} \bullet \text{B} \bullet \text{C} \bullet\)

**Solution**

Postulate 8 Through any three [noncollinear](#) points there exists exactly one plane.
Use the diagram to write examples of Postulates 9 and 11.

Postulate 9  Plane \( Q \) contains at least three noncollinear points, \( W, V, \) and \( Y \).

Postulate 11  The intersection of plane \( P \) and plane \( Q \) is \( \text{line } b \).

Checkpoint  Use the diagram in Example 2 to complete the following exercises.

1. Which postulate allows you to say that the intersection of line \( a \) and line \( b \) is a point?
   Postulate 7

2. Write examples of Postulates 5 and 6.
   Line \( a \) passes through \( X \) and \( Y \); line \( a \) contains points \( X \) and \( Y \).

CONCEPT SUMMARY: INTERPRETING A DIAGRAM

When you interpret a diagram, you can only assume information about size or measure if it is marked.

YOU CAN ASSUME

All points shown are \text{coplanar}.
\( \angle AHB \) and \( \angle BHD \) are a linear pair.
\( \angle AHF \) and \( \angle BHD \) are vertical angles.
\( A, H, J, \) and \( D \) are \text{collinear}.
\( \overrightarrow{AD} \) and \( \overrightarrow{BF} \) intersect at \( H \).

YOU CANNOT ASSUME

\( G, F, \) and \( E \) are collinear.
\( \overrightarrow{BF} \) and \( \overrightarrow{CE} \) intersect.
\( \overrightarrow{BF} \) and \( \overrightarrow{CE} \) do not intersect.
\( \angle BHA \not\cong \angle CJA \)
\( \overrightarrow{AD} \perp \overrightarrow{BF} \) or \( m \angle AHB = 90^\circ \)
Example 3  Use given information to sketch a diagram

Sketch a diagram showing RS perpendicular to TV, intersecting at point X.

Solution
Step 1  Draw RS and label points R and S.
Step 2  Draw a point X between R and S.
Step 3  Draw TV through X so that it is perpendicular to RS.

Example 4  Interpret a diagram in three dimensions

Which of the following statements cannot be assumed from the diagram?

E, D, and C are collinear.
The intersection of BD and EC is D.
\( \overrightarrow{BD} \perp \overrightarrow{EC} \)
\( \overrightarrow{EC} \perp \text{plane } G \)

Solution
With no right angles marked, you cannot assume that \( \overrightarrow{BD} \perp \overrightarrow{EC} \) or \( \overrightarrow{EC} \perp \text{plane } G \).

✔ Checkpoint  Complete the following exercises.

3. In Example 3, if the given information indicated that RX and XS are congruent, how would the diagram change?

   Point X would be drawn as the midpoint of RS and the congruent segments would be marked.

4. In the diagram for Example 4, can you assume that BD is the intersection of plane F and plane G?

   Yes
## 2.5 Reason Using Properties from Algebra

**Goal**
- Use algebraic properties in logical arguments.

### Your Notes

#### ALGEBRAIC PROPERTIES OF EQUALITY

Let \( a, b, \) and \( c \) be real numbers.

- **Addition Property**
  
  If \( a = b \), then \( a + c = b + c \).

- **Subtraction Property**
  
  If \( a = b \), then \( a - c = b - c \).

- **Multiplication Property**
  
  If \( a = b \), then \( ac = bc \).

- **Division Property**
  
  If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).

- **Substitution Property**
  
  If \( a = b \), then \( a \) can be substituted for \( b \) in any equation or expression.

### Example 1

**Write reasons for each step**

Solve \(2x + 3 = 9 - x\). Write a reason for each step.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x + 3 = 9 - x)</td>
<td>Write original equation.</td>
<td>Given</td>
</tr>
<tr>
<td>(2x + 3 + x = 9 - x + x)</td>
<td>Add (x) to each side.</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>(3x + 3 = 9)</td>
<td>Combine like terms.</td>
<td>Simplify.</td>
</tr>
<tr>
<td>(3x = 6)</td>
<td>Subtract (3) from each side.</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>(x = 2)</td>
<td>Divide each side by (3).</td>
<td>Division Property of Equality</td>
</tr>
</tbody>
</table>

The value of \(x\) is \(2\).
DISTRIBUTIVE PROPERTY

\[ a(b + c) = ab + ac \], where \( a, b, \) and \( c \) are real numbers.

Example 2  Use the Distributive Property

Solve \(-4(6x + 2) = 64\). Write a reason for each step.

Solution

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4(6x + 2) = 64)</td>
<td>Write original equation.</td>
<td>Given</td>
</tr>
<tr>
<td>(-24x - 8 = 64)</td>
<td>Multiply.</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>(-24x = 72)</td>
<td>Add 8 to each side.</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>(x = -3)</td>
<td>Divide each side by (-24).</td>
<td>Division Property of Equality</td>
</tr>
</tbody>
</table>

Checkpoint  Complete the following exercises.

1. Solve \(x - 5 = 7 + 2x\). Write a reason for each step.

\[
\begin{align*}
x - 5 &= 7 + 2x & \text{Given} \\
x - 5 - x &= 7 + 2x - x & \text{Subtraction Property of Equality} \\
-5 &= 7 + x & \text{Simplify.} \\
-12 &= x & \text{Subtraction Property of Equality}
\end{align*}
\]

2. Solve \(4(5 - x) = -2x\). Write a reason for each step.

\[
\begin{align*}
4(5 - x) &= -2x & \text{Given} \\
20 - 4x &= -2x & \text{Distributive Property} \\
20 &= 2x & \text{Addition Property of Equality} \\
10 &= x & \text{Division Property of Equality}
\end{align*}
\]
Example 3  Use properties in the real world

Speed  A motorist travels 5 miles per hour slower than the speed limit $s$ for 3.5 hours. The distance traveled $d$ can be determined by the formula $d = 3.5(s - 5)$. Solve for $s$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 3.5(s - 5)$</td>
<td>Write original equation.</td>
<td>Given</td>
</tr>
<tr>
<td>$d = 3.5s - 17.5$</td>
<td>Multiply.</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>$d + 17.5 = 3.5s$</td>
<td>Add 17.5 to each side.</td>
<td>Addition</td>
</tr>
<tr>
<td>$d + \frac{17.5}{3.5} = s$</td>
<td>Divide each side by 3.5.</td>
<td>Division</td>
</tr>
</tbody>
</table>

REFLEXIVE PROPERTY OF EQUALITY
Real Numbers  For any real number $a$, $a = a$.
Segment Length  For any segment $AB$, $AB = AB$.
Angle Measure  For any angle $A$, $m\angle A = m\angle A$.

SYMMETRIC PROPERTY OF EQUALITY
Real Numbers  For any real numbers $a$ and $b$, if $a = b$, then $b = a$.
Segment Length  For any segments $AB$ and $CD$, if $AB = CD$, then $CD = AB$.
Angle Measure  For any angles $A$ and $B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

TRANSITIVE PROPERTY OF EQUALITY
Real Numbers  For any real numbers $a$, $b$, and $c$, if $a = b$ and $b = c$, then $a = c$.
Segment Length  For any segments $AB$, $CD$, and $EF$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure  For any angles $A$, $B$, and $C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$. 
Show that $CF = AD$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB = EF$</td>
<td>Given</td>
</tr>
<tr>
<td>$BC = DE$</td>
<td>Given</td>
</tr>
<tr>
<td>$AC = AB + BC$</td>
<td>Segment Addition Postulate</td>
</tr>
<tr>
<td>$DF = DE + EF$</td>
<td>Segment Addition Postulate</td>
</tr>
<tr>
<td>$DF = BC + AB$</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>$DF = AC$</td>
<td>Transitive Property of Equality</td>
</tr>
<tr>
<td>$DF + CD = AC + CD$</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>$CF = AD$</td>
<td>Substitution Property of Equality</td>
</tr>
</tbody>
</table>

✓ Checkpoint Complete the following exercises. In Exercises 4–6, name the property of equality that the statement illustrates.

3. Suppose the equation in Example 3 is $d = 5(s + 3)$. Solve for $s$. Write a reason for each step.

\[
\begin{align*}
    d &= 5(s + 3) \quad \text{Given} \\
    d &= 5s + 15 \quad \text{Distributive Property} \\
    d - 15 &= 5s \quad \text{Subtraction Property of Equality} \\
    \frac{d - 15}{5} &= s \quad \text{Division Property of Equality}
\end{align*}
\]

4. If $GH = JK$, then $JK = GH$.

    Symmetric Property of Equality for Segment Length

5. If $r = s$, and $s = 44$, then $r = 44$.

    Transitive Property of Equality for Real Numbers

6. $m\angle N = m\angle N$

    Reflexive Property of Equality for Angle Measure
2.6 Prove Statements about Segments and Angles

Goal • Write proofs using geometric theorems.

Your Notes

VOCABULARY

Proof A proof is a logical argument that shows a statement is true.

Two-column proof A two-column proof has numbered statements and corresponding reasons that show an argument in logical order.

Theorem A theorem is a statement that can be proven.

Example 1 Write a two-column proof

Use the diagram to prove $m\angle 1 = m\angle 4$.  
Given $m\angle 2 = m\angle 3$, $m\angle AXD = m\angle AXC$  
Prove $m\angle 1 = m\angle 4$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m\angle AXC = m\angle AXD$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle AXD$</td>
<td>2. Angle Addition Postulate</td>
</tr>
<tr>
<td>= $m\angle 1 + m\angle 2$</td>
<td></td>
</tr>
<tr>
<td>3. $m\angle AXC$</td>
<td>3. Angle Addition Postulate</td>
</tr>
<tr>
<td>= $m\angle 3 + m\angle 4$</td>
<td></td>
</tr>
<tr>
<td>4. $m\angle 1 + m\angle 2$</td>
<td>4. Substitution Property of Equality</td>
</tr>
<tr>
<td>= $m\angle 3 + m\angle 4$</td>
<td></td>
</tr>
<tr>
<td>5. $m\angle 2 = m\angle 3$</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. $m\angle 1 + m\angle 3$</td>
<td>6. Substitution Property of Equality</td>
</tr>
<tr>
<td>= $m\angle 3 + m\angle 4$</td>
<td></td>
</tr>
<tr>
<td>7. $m\angle 1 = m\angle 4$</td>
<td>7. Subtraction Property of Equality</td>
</tr>
</tbody>
</table>
**THEOREM 2.1  CONGRUENCE OF SEGMENTS**

Segment congruence is reflexive, symmetric, and transitive.

Reflexive  
For any segment $AB$, $\overline{AB} \cong \overline{AB}$.

Symmetric  
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive  
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

**THEOREM 2.2  CONGRUENCE OF ANGLES**

Angle congruence is reflexive, symmetric, and transitive.

Reflexive  
For any angle $\angle A$, $\angle A \cong \angle A$.

Symmetric  
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive  
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

---

**Example 2  Name the property shown**

Name the property illustrated by the statement.

If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$.

**Symmetric Property of Angle Congruence**

---

**Checkpoint  Complete the following exercises.**

1. Three steps of a proof are shown. Give the reasons for the last two steps.

   **Given**  $BC = AB$

   **Prove**  $AC = AB + AB$

   **Statements**
   
   1. $BC = AB$  
   2. $AC = AB + BC$  
   3. $AC = AB + AB$

   **Reasons**
   
   1. Given  
   2. **Segment Addition Postulate**  
   3. **Substitution Property of Equality**

2. Name the property illustrated by the statement.
   If $\angle H \cong \angle T$ and $\angle T \cong \angle B$, then $\angle H \cong \angle B$.

   **Transitive Property of Angle Congruence**
Example 3  Use properties of equality

If you know that \( \overrightarrow{BD} \) bisects \( \angle ABC \), prove that \( m\angle ABC \) is two times \( m\angle 1 \).

Given \( \overrightarrow{BD} \) bisects \( \angle ABC \).
Prove \( m\angle ABC = 2 \cdot m\angle 1 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overrightarrow{BD} ) bisects ( \angle ABC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \equiv \angle 2 )</td>
<td>2. Definition of angle bisector</td>
</tr>
<tr>
<td>3. ( m\angle 1 = m\angle 2 )</td>
<td>3. Definition of congruent angles</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = m\angle ABC )</td>
<td>4. Angle Addition Postulate</td>
</tr>
<tr>
<td>5. ( m\angle 1 + m\angle 1 = m\angle ABC )</td>
<td>5. Substitution Property of Equality</td>
</tr>
<tr>
<td>6. ( 2 \cdot m\angle 1 = m\angle ABC )</td>
<td>6. Distributive Property</td>
</tr>
</tbody>
</table>

CONCEPT SUMMARY: WRITING A TWO-COLUMN PROOF

Proof of the Symmetric Property of Segment Congruence

Given \( \overline{AB} \equiv \overline{CD} \)
Prove \( \overline{CD} \equiv \overline{AB} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \equiv \overline{CD} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{AB} = \overline{CD} )</td>
<td>2. Definition of congruent segments</td>
</tr>
<tr>
<td>3. ( \overline{CD} = \overline{AB} )</td>
<td>3. Symmetric Property of Equality</td>
</tr>
<tr>
<td>4. ( \overline{CD} \equiv \overline{AB} )</td>
<td>4. Definition of congruent segments</td>
</tr>
</tbody>
</table>

The number of statements will vary. Remember to give a reason for the last statement. Definitions, postulates, or proven theorems that allow you to state the corresponding statement.
Interstate There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and the Mason exit. The distance between rest area B and the Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

Solution

Step 1 Draw a diagram.

Step 2 Draw diagrams showing relationships.

Step 3 Write a proof.

Given \( R \) is the midpoint of \( \overline{AM} \), \( MB = AR \).
Prove \( M \) is the midpoint of \( \overline{RB} \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( R ) is the midpoint of ( \overline{AM} ), ( MB = AR ).</td>
<td>1. <strong>Given</strong></td>
</tr>
<tr>
<td>2. ( AR \cong RM )</td>
<td>2. Definition of midpoint</td>
</tr>
<tr>
<td>3. ( AR = RM )</td>
<td>3. Definition of congruent segments</td>
</tr>
<tr>
<td>4. ( MB = RM )</td>
<td>4. Transitive Property of Congruence</td>
</tr>
<tr>
<td>5. ( MB \cong RM )</td>
<td>5. Definition of congruent segments</td>
</tr>
<tr>
<td>6. ( M ) is the midpoint of ( \overline{RB} ).</td>
<td>6. Definition of midpoint</td>
</tr>
</tbody>
</table>

Checkpoint Complete the following exercise.

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

Rest area A and rest area B
2.7 Prove Angle Pair Relationships

**Goal**
- Use properties of special pairs of angles.

**THEOREM 2.3 RIGHT ANGLES CONGRUENCE THEOREM**

All right angles are congruent.

**THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM**

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If \( \angle 1 \) and \( \angle 2 \) are supplementary and \( \angle 3 \) and \( \angle 2 \) are supplementary, then \( \angle 1 \equiv \angle 3 \).

**THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM**

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If \( \angle 4 \) and \( \angle 5 \) are complementary and \( \angle 6 \) and \( \angle 5 \) are complementary, then \( \angle 4 \equiv \angle 6 \).

**Example 1**

Use right angle congruence

Write a proof.

**Given** \( JK \perp KL, ML \perp KL \)

**Prove** \( \angle K \equiv \angle L \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( JK \perp KL, ML \perp KL )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle K ) and ( \angle L ) are right angles.</td>
<td>2. Definition of perpendicular lines</td>
</tr>
<tr>
<td>3. ( \angle K \equiv \angle L )</td>
<td>3. Right Angles Congruence Theorem</td>
</tr>
</tbody>
</table>

The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show that the angles are right angles.
Example 2  Use the Congruent Supplements Theorem

Write a proof.

Given \( \angle 1 \) and \( \angle 2 \) are supplements.
\( \angle 1 \) and \( \angle 4 \) are supplements.
\( m\angle 2 = 45^\circ \)

Prove \( m\angle 4 = 45^\circ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 ) and ( \angle 2 ) are supplements. ( \angle 1 ) and ( \angle 4 ) are supplements.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 2 \cong \angle 4 )</td>
<td>2. Congruent Supplements Theorem</td>
</tr>
<tr>
<td>3. ( m\angle 2 = m\angle 4 )</td>
<td>3. Definition of congruent angles</td>
</tr>
<tr>
<td>4. ( m\angle 2 = 45^\circ )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( m\angle 4 = 45^\circ )</td>
<td>5. Substitution Property of Equality</td>
</tr>
</tbody>
</table>

Checkpoint Complete the following exercises.

1. In Example 1, suppose you are given that \( \angle K \cong \angle L \). Can you use the Right Angles Congruence Theorem to prove that \( \angle K \) and \( \angle L \) are right angles? Explain.

   No, you cannot prove that \( \angle K \) and \( \angle L \) are right angles, because the converse of the Right Angles Congruence Theorem is not always true.

2. Suppose \( \angle A \) and \( \angle B \) are complements, and \( \angle A \) and \( \angle C \) are complements. Can \( \angle B \) and \( \angle C \) be supplements? Explain.

   No, \( \angle B \) and \( \angle C \) are complements by the Congruent Complements Theorem, so they cannot be supplements.
POSTULATE 12  LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are supplementary. ∠1 and ∠2 form a linear pair, so ∠1 and ∠2 are supplementary and \( m\angle 1 + m\angle 2 = 180^\circ \).

THEOREM 2.6  VERTICAL ANGLES CONGRUENCE THEOREM

Vertical angles are congruent.

Example 3  Determine whether a proof is valid.

Tell if the proof is valid, or explain how to make it valid.

Given ∠4 is a right angle.

Prove ∠2 and ∠4 are supplementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠4 is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 4 = 90^\circ )</td>
<td>2. Definition of a right angle</td>
</tr>
<tr>
<td>3. ( m\angle 2 = 90^\circ )</td>
<td>3. Vertical Angles Congruence Theorem</td>
</tr>
<tr>
<td>4. ∠2 and ∠4 are supplementary.</td>
<td>4. ( m\angle 2 + m\angle 4 = 180^\circ )</td>
</tr>
</tbody>
</table>

The proof is not valid. The Vertical Angle Congruence Theorem proves that \( \angle 2 \cong \angle 4 \). It does not let you immediately conclude that \( m\angle 2 = 90^\circ \). Here is a valid proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠4 is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 4 = 90^\circ )</td>
<td>2. Definition of a right angle</td>
</tr>
<tr>
<td>3. ( \angle 2 \cong \angle 4 )</td>
<td>3. Vertical Angles Congruence Theorem</td>
</tr>
<tr>
<td>4. ( m\angle 2 = m\angle 4 )</td>
<td>4. Definition of ( \cong ) angles</td>
</tr>
<tr>
<td>5. ( m\angle 2 = 90^\circ )</td>
<td>5. Substitution Property of Equality</td>
</tr>
<tr>
<td>6. ∠2 and ∠4 are supplementary.</td>
<td>6. ( m\angle 2 + m\angle 4 = 180^\circ )</td>
</tr>
</tbody>
</table>
Find $m \angle FKG$.

**Solution**

Because $m \angle FKG$ and $m \angle GKH$ form a linear pair, the sum of their measures is $180^\circ$.

\[
(4x - 1)^\circ + 113^\circ = 180^\circ
\]

Write equation.

\[
4x + 112 = 180
\]

Simplify.

\[
x = 68
\]

Subtract.

\[
x = 17
\]

Divide.

Use $x = 17$ to find $m \angle FKG$.

\[
m \angle FKG = (4x - 1)^\circ
\]

Write equation.

\[
= [4(17) - 1]^\circ
\]

Substitute 17 for $x$.

\[
= 68 - 1
\]

Multiply.

\[
= 67^\circ
\]

Simplify.

The measure of $\angle FKG$ is $67^\circ$.

**Checkpoint**  Complete the following exercises.

3. If $m \angle 4 = 63^\circ$, find $m \angle 1$ and $m \angle 2$.

\[
m \angle 1 = 117^\circ, m \angle 2 = 63^\circ
\]

4. Explain why the definition of congruent angles is necessary for the proof in Example 3 to be logically valid.

**Sample answer:** The Vertical Angles Congruence Theorem only proves that the angles are congruent. The definition of congruent angles is necessary to prove that their measures are equal.

5. Find $m \angle AEB$.

\[
m \angle AEB = 70^\circ
\]
Words to Review

Give an example of the vocabulary word.

<table>
<thead>
<tr>
<th>Conjecture</th>
<th>Inductive reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A conjecture is an unproven statement that is based on observations.</td>
<td>You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counterexample</th>
<th>Conditional statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A counterexample is a specific case for which the conjecture is false.</td>
<td>Leaves change color in fall.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If-then form</th>
<th>Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>If leaves are changing color, then it is fall.</td>
<td>If leaves are changing color . . .</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conclusion</th>
<th>Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>. . . then it is fall.</td>
<td>The leaves are not changing color.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Converse</th>
<th>Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>If it is fall, then leaves are changing color.</td>
<td>If leaves are not changing color, then it is not fall.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contrapositive</th>
<th>Equivalent statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>If it is not fall, then leaves are not changing color.</td>
<td>A conditional statement is equivalent to its contrapositive. The inverse and converse of a conditional statement are also equivalent.</td>
</tr>
<tr>
<td>Perpendicular lines</td>
<td>Biconditional statement</td>
</tr>
<tr>
<td>----------------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td><img src="image" alt="Perpendicular lines" /></td>
<td>The value of $x$ is 5 if and only if $x - 3 = 2$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deductive reasoning</th>
<th>Logically equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.</td>
<td>If $p$, then $q$. If not $q$, then not $p$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Truth table/Truth values</th>
<th>Line perpendicular to a plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proof</th>
<th>Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>A logical argument that shows a statement is true.</td>
<td>Vertical angles are congruent.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Two-column proof</th>
</tr>
</thead>
</table>

Given $\angle 1 \cong \angle 2$, $m\angle 1 = 60^\circ$
Prove $m\angle 2 = 60^\circ$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle 1 = m\angle 2$</td>
<td>2. Definition of congruent $\triangle$</td>
</tr>
<tr>
<td>3. $m\angle 1 = 60^\circ$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $m\angle 2 = 60^\circ$</td>
<td>4. Transitive Property of $=$</td>
</tr>
</tbody>
</table>

Review your notes and Chapter 2 by using the Chapter Review on pages 137–140 of your textbook.
3.1 Identify Pairs of Lines and Angles

**Goal** • Identify angle pairs formed by three intersecting lines.

**Your Notes**

<table>
<thead>
<tr>
<th>VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parallel lines</strong> Two lines are parallel lines if they do not intersect and are coplanar.</td>
</tr>
<tr>
<td><strong>Skew lines</strong> Two lines are skew lines if they do not intersect and are not coplanar.</td>
</tr>
<tr>
<td><strong>Parallel planes</strong> Two planes that do not intersect are parallel planes.</td>
</tr>
<tr>
<td><strong>Transversal</strong> A transversal is a line that intersects two or more coplanar lines at different points.</td>
</tr>
<tr>
<td><strong>Corresponding angles</strong> Two angles are corresponding angles if they have corresponding positions.</td>
</tr>
<tr>
<td><strong>Alternate interior angles</strong> Two angles are alternate interior angles if they lie between the two lines and on opposite sides of the transversal.</td>
</tr>
<tr>
<td><strong>Alternate exterior angles</strong> Two angles are alternate exterior angles if they lie outside the two lines and on opposite sides of the transversal.</td>
</tr>
<tr>
<td><strong>Consecutive interior angles</strong> Two angles are consecutive interior angles if they lie between the two lines and on the same side of the transversal.</td>
</tr>
</tbody>
</table>
Example 1: Identify relationships in space

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?

a. Line(s) parallel to \( \overrightarrow{AF} \) and containing point \( E \)
   - Solution:
   - \( \overrightarrow{EJ}, \overrightarrow{BG}, \overrightarrow{CH}, \) and \( \overrightarrow{DI} \) all appear parallel to \( \overrightarrow{AF} \), but only \( \overrightarrow{EJ} \) contains point \( E \).

b. Line(s) skew to \( \overrightarrow{AF} \) and containing point \( E \)
   - Solution:
   - \( \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{GH}, \overrightarrow{HI}, \) and \( \overrightarrow{IJ} \) all appear skew to \( \overrightarrow{AF} \), but only \( \overrightarrow{DE} \) contains point \( E \).

c. Line(s) perpendicular to \( \overrightarrow{AF} \) and containing point \( E \)
   - Solution:
   - \( \overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{FG}, \) and \( \overrightarrow{FJ} \) all appear perpendicular to \( \overrightarrow{AF} \), but only \( \overrightarrow{AE} \) contains point \( E \).

d. Plane(s) parallel to plane \( FGH \) and containing point \( E \)
   - Solution:
   - Plane \( \overrightarrow{ABC} \) appears parallel to plane \( FGH \) and contains point \( E \).

Checkpoint: Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?

1. parallel to \( \overrightarrow{MN} \) and contains \( J \)
   - Solution:
   - \( \overrightarrow{JL} \)

2. skew to \( \overrightarrow{MN} \) and contains \( J \)
   - Solution:
   - \( \overrightarrow{KJ} \)

3. perpendicular to \( \overrightarrow{MN} \) and contains \( J \)
   - Solution:
   - \( \overrightarrow{JN} \)

4. Name the plane that contains \( J \) and appears to be parallel to plane \( MNO \).
   - Solution:
   - Plane \( JKL \)
Your Notes

**POSTULATE 13  PARALLEL POSTULATE**

If there is a line and a point not on the line, then there is **exactly one** line through the point parallel to the given line.

There is exactly one line through \( P \) parallel to \( \ell \).

---

**POSTULATE 14  PERPENDICULAR POSTULATE**

If there is a line and a point not on the line, then there is **exactly one** line through the point perpendicular to the given line.

There is exactly one line through \( P \) perpendicular to \( \ell \).

---

**Example 2**

*Identify parallel and perpendicular lines*

Use the diagram at the right to answer each question.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is \( \overrightarrow{AB} \perp \overrightarrow{BC} \)? Explain.

**Solution**

a. \( \overrightarrow{AB} \parallel \overrightarrow{CD} \)

b. \( \overrightarrow{AB} \perp \overrightarrow{AC} \)

c. \( \overrightarrow{AB} \) is not perpendicular to \( \overrightarrow{BC} \), because \( \overrightarrow{AB} \) is perpendicular to \( \overrightarrow{AC} \) and by the **Perpendicular Postulate** there is exactly one line perpendicular to \( \overrightarrow{AB} \) through \( C \).

**Checkpoint** Complete the following exercise.

5. In Example 2, can you use the **Perpendicular Postulate** to show that \( \overrightarrow{AC} \perp \overrightarrow{CD} \)? Explain.

   No, there is no right angle symbol at \( C \) so you do not know if \( \overrightarrow{AC} \perp \overrightarrow{CD} \).
ANGLES FORMED BY TRANSVERSALS

Two angles are **corresponding** angles if they have corresponding positions. For example, \( \angle 2 \) and \( \angle 6 \) are above the lines and to the right of the transversal \( t \).

Two angles are **alternate interior** angles if they lie between the two lines and on opposite sides of the transversal.

Two angles are **alternate exterior** angles if they lie outside the two lines and on opposite sides of the transversal.

Two angles are **consecutive interior** angles if they lie between the two lines and on the same side of the transversal.

Another name for consecutive interior angles is **same-side interior angles**.

Example 3

Identify all pairs of (a) corresponding angles, (b) alternate interior angles, (c) alternate exterior angles, and (d) consecutive interior angles.

a. \( \angle 1 \) and \( \angle 3 \), \( \angle 2 \) and \( \angle 4 \), \( \angle 5 \) and \( \angle 7 \), \( \angle 6 \) and \( \angle 8 \)

b. \( \angle 2 \) and \( \angle 7 \), \( \angle 6 \) and \( \angle 3 \)

c. \( \angle 5 \) and \( \angle 4 \), \( \angle 1 \) and \( \angle 8 \)

d. \( \angle 2 \) and \( \angle 3 \), \( \angle 6 \) and \( \angle 7 \)

Checkpoint

Classify the pair of numbered angles.

6. Consecutive interior angles

7. Alternate exterior angles
**3.2 Use Parallel Lines and Transversals**

**Goal**
- Use angles formed by parallel lines and transversals.

**Your Notes**

**POSTULATE 15** **CORRESPONDING ANGLES POSTULATE**

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are _congruent_.

![Diagram showing corresponding angles]

**Example 1** **Identify congruent angles**

The measure of three of the numbered angles is $125^\circ$. Identify the angles. _Explain_ your reasoning.

**Solution**

By the Corresponding Angles Postulate, $m\angle 7 = 125^\circ$.

Using the Vertical Angles Congruence Theorem, $m\angle 1 = 125^\circ$.

Because $\angle 1$ and $\angle 5$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 5 = 125^\circ$.

**Checkpoint** Complete the following exercise using the diagram shown.

1. If $m\angle 7 = 75^\circ$, find $m\angle 1, m\angle 3,$ and $m\angle 5$. Tell which postulate or theorem you use in each case.

   - $m\angle 3 = 75^\circ$, Corresponding Angles Postulate;
   - $m\angle 5 = 75^\circ$, Vertical Angles Congruence Theorem;
   - $m\angle 1 = 75^\circ$, Corresponding Angles Postulate
**THEOREM 3.1** ALTERNATE INTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are **congruent**.

**THEOREM 3.2** ALTERNATE EXTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are **congruent**.

**THEOREM 3.3** CONSECUTIVE INTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are **supplementary**.

---

**Example 2** Use properties of parallel lines

Find the value of $x$.

**Solution**

Lines $r$ and $s$ are **parallel**, so you can use the theorems about parallel lines.

$113^\circ = (3x - 4)^\circ$  \hspace{1cm} \text{Alternate Exterior Angles Theorem}

$117 = 3x$  \hspace{1cm} \text{Add 4 to each side.}

$39 = x$  \hspace{1cm} \text{Divide each side by 3.}

The value of $x$ is **39**.
Runways  A taxiway is being constructed that intersects two parallel runways at an airport. You know that \( m \angle 2 = 98^\circ \). What is \( m \angle 1 \)? How do you know?

**Solution**
Because the runways are parallel, \( \angle 1 \) and \( \angle 2 \) are alternate interior angles. By the Alternate Interior Angles Theorem, \( \angle 1 \equiv \angle 2 \). By the definition of congruent angles, \( m \angle 1 = m \angle 2 = 98^\circ \).

**Checkpoint** Complete the following exercises.

2. Find the value of \( x \).

\[
(3x - 7)^\circ \quad (x + 5)^\circ
\]

\( x = 45.5 \)

3. In Example 3, suppose \( \angle 3 \) is the consecutive interior angle with \( \angle 2 \). What is \( m \angle 3 \)?

\( 82^\circ \)


**VOCABULARY**

Paragraph proof  A proof can be written in paragraph form, called a paragraph proof.

**POSTULATE 16  CORRESPONDING ANGLES CONVERSE**

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

---

**Example 1  Apply the Corresponding Angles Converse**

Find the value of $x$ that makes $m \parallel n$.

**Solution**

Lines $m$ and $n$ are parallel if the marked corresponding angles are congruent.

$(2x + 3)° = 71°$  Use Postulate 16 to write an equation.

$2x = 68$  Subtract 3 from each side.

$x = 34$  Divide each side by 2.

The lines $m$ and $n$ are parallel when $x = 34$.

---

**Checkpoint** Find the value of $x$ that makes $a \parallel b$.

1. $x = 21$
THEOREM 3.4  ALTERNATE INTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are \textit{parallel}.

THEOREM 3.5  ALTERNATE EXTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are \textit{parallel}.

THEOREM 3.6  CONSECUTIVE INTERIOR ANGLES CONVERSE
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are \textit{parallel}.

Example 2  \textbf{Solve a real-world problem}
Flags  How can you tell whether the sides of the flag of Nepal are parallel?

Solution
Because the alternate interior angles are congruent, you know that the sides of the flag are \textit{parallel}.

_checkpoint Can you prove that lines $a$ and $b$ are parallel? \textit{Explain} why or why not.

2. $m\angle 1 + m\angle 2 = 180^\circ$

Yes, you can use the Consecutive Interior Angles Converse to prove $a \parallel b$. 
Example 3  **Write a paragraph proof**

In the figure, $a \parallel b$ and $\angle 1$ is congruent to $\angle 3$. Prove $x \parallel y$.

**Solution**

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

**Plan for Proof**

a. Look at $\angle 1$ and $\angle 2$.

b. Look at $\angle 2$ and $\angle 3$.

**Plan in Action**

a. It is given that $a \parallel b$, so by the **Corresponding Angles Postulate**, $\angle 1 \equiv \angle 2$.

b. It is also given that $\angle 1 \equiv \angle 3$. Then $\angle 2 \equiv \angle 3$ by the Transitive Property of Congruence for angles. Therefore, by the **Alternate Exterior Angles Converse**, $x \parallel y$.

**Checkpoint**  Complete the following exercise.

3. In Example 3, suppose it is given that $\angle 1 \equiv \angle 3$ and $x \parallel y$. Complete the following paragraph proof showing that $a \parallel b$.

   It is given that $x \parallel y$. By the Exterior Angles Postulate, $\angle 2 \equiv \angle 3$.

   It is also given that $\angle 1 \equiv \angle 3$. Then $\angle 1 \equiv \angle 2$ by the Transitive Property of Congruence for angles. Therefore, by the **Corresponding Angles Converse**, $a \parallel b$.
Lesson 3.3 • Geometry Notetaking Guide

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Lesson 3.3

**Your Notes**

**THEOREM 3.7 TRANSITIVE PROPERTY OF PARALLEL LINES**

If two lines are parallel to the same line, then they are parallel to each other.

![Diagram of parallel lines](image)

If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

**Example 4 Use the Transitive Property of Parallel Lines**

Utility poles Each utility pole shown is parallel to the pole immediately to its right. Explain why the leftmost pole is parallel to the rightmost pole.

**Solution**

The poles from left to right can be named $t_1, t_2, t_3, \ldots, t_6$. Each pole is parallel to the one to its right, so $t_1 \parallel t_2$, $t_2 \parallel t_3$, and so on. Then $t_1 \parallel t_3$ by the **Transitive Property of Parallel Lines**. Similarly, because $t_3 \parallel t_4$, it follows that $t_1 \parallel t_4$. By continuing this reasoning, $t_1 \parallel t_6$. So, the leftmost pole is parallel to the rightmost pole.

**Checkpoint** Complete the following exercise.

4. Each horizontal piece of the window blinds shown is called a slat. Each slat is parallel to the slat immediately below it. Explain why the top slat is parallel to the bottom slat.

The slats from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{16}$. Each slat is parallel to the one below it, so $s_1 \parallel s_2, s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{16}$. So, the top slat is parallel to the bottom slat.

**Homework**

- Complete the following exercise.

---

When you name several similar items, you can use one variable with subscripts to keep track of the items.

---
3.4 Find and Use Slopes of Lines

Goal • Find and compare slopes of lines.

VOCABULARY
Slope The slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line.

SLOPE OF LINES IN THE COORDINATE PLANE
Negative slope: falls from left to right, as in line j
Positive slope: rises from left to right, as in line k
Undefined slope: vertical, as in line n
Zero slope (slope of 0): horizontal, as in line l

Example 1 Find slopes of lines in a coordinate plane
Find the slope of line a and line c.

Slope of line a:
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{4 - 0} = \frac{4}{4} = 1 \]

Slope of line c:
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{4 - 0} = \frac{6}{4} = 0 \]

Checkpoint Use the graph in Example 1. Find the slope of the line.

1. line b
   -1

2. line d
   undefined
POSTULATE 17  SLOPES OF PARALLEL LINES
In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.
Any two vertical lines are parallel.

POSTULATE 18  SLOPES OF PERPENDICULAR LINES
In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.
Horizontal lines are perpendicular to vertical lines.

Example 2  Identify parallel lines
Find the slope of each line.
Which lines are parallel?

Solution
Find the slope of $k_1$.
\[
m = \frac{-1 - (-6)}{-3 - (-4)} = \frac{5}{1} = 5
\]

Find the slope of $k_2$.
\[
m = \frac{2 - (-4)}{2 - 1} = 6
\]

Find the slope of $k_3$.
\[
m = \frac{2 - (-3)}{4 - 3} = 5
\]

Compare the slopes. Because $k_1$ and $k_3$ have the same slope, they are parallel. The slope of $k_2$ is different, so $k_2$ is not parallel to the other lines.

Checkpoint  Complete the following exercise.

3. Line $c$ passes through $(2, -2)$ and $(5, 7)$. Line $d$ passes through $(-3, 4)$ and $(1, -8)$. Are the two lines parallel? Explain how you know.

No; the slope of $c$ is not equal to the slope of $d$. 
**Example 3**  \textit{Draw a perpendicular line}

Line \( h \) passes through \((1, -2)\) and \((5, 6)\). Graph the line perpendicular to \( h \) that passes through the point \((2, 5)\).

\textbf{Step 1} Find the slope \( m_1 \) of \( h \) through \((1, -2)\) and \((5, 6)\).

\[
m_1 = \frac{6 - (-2)}{5 - 1} = \frac{8}{4} = 2
\]

\textbf{Step 2} Find the slope \( m_2 \) of a line perpendicular to \( h \).

\[
2 \cdot m_2 = -1
\]

\[
m_2 = -\frac{1}{2}
\]

\textbf{Step 3} Use the rise and run to graph the line.

**Example 4**  \textit{Use properties of a quadrilateral}

Quadrilateral \( ABCD \) has vertices \( A(1, 5) \), \( B(6, 7) \), \( C(8, 2) \), and \( D(3, 0) \). Is \( ABCD \) a rectangle, a square, or neither?

<table>
<thead>
<tr>
<th>Line</th>
<th>Find Slopes</th>
<th>Find Lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB )</td>
<td>( \frac{7 - 5}{6 - 1} = \frac{2}{5} )</td>
<td>( \sqrt{(6 - 1)^2 + (7 - 5)^2} = \sqrt{29} )</td>
</tr>
<tr>
<td>( BC )</td>
<td>( \frac{2 - 7}{8 - 6} = -\frac{5}{2} )</td>
<td>( \sqrt{(8 - 6)^2 + (2 - 7)^2} = \sqrt{29} )</td>
</tr>
<tr>
<td>( CD )</td>
<td>( \frac{0 - 2}{3 - 8} = \frac{2}{5} )</td>
<td>( \sqrt{(3 - 8)^2 + (0 - 2)^2} = \sqrt{29} )</td>
</tr>
<tr>
<td>( DA )</td>
<td>( \frac{5 - 0}{1 - 3} = -\frac{5}{2} )</td>
<td>( \sqrt{(1 - 3)^2 + (5 - 0)^2} = \sqrt{29} )</td>
</tr>
</tbody>
</table>

Multiplying slopes of consecutive sides yields \( -1 \), so \( ABCD \) has four right angles. The four sides are all the same length, so \( ABCD \) is a square.

**Checkpoint** Complete the following exercise

4. Quadrilateral \( ABCD \) has vertices \( A(-1, 1) \), \( B(4, 3) \), \( C(6, -2) \), \( D(1, -4) \). Show it has congruent diagonals.

\[
AC = BD = \sqrt{58}, \text{ so } AC \cong BD.
\]
Write an equation of the line in slope-intercept form.

Solution

Step 1 Find the slope. Choose two points on the graph of the line, (0, 3) and (2, −1).

\[ m = \frac{3 - (-1)}{0 - 2} = \frac{4}{-2} = -2 \]

Step 2 Find the y-intercept. The line intersects the y-axis at the point (0, 3), so the y-intercept is 3.

Step 3 Write the equation.

\[ y = mx + b \]

Use slope-intercept form.

\[ y = -2x + 3 \]

Substitute −2 for \( m \) and 3 for \( b \).
Write an equation of the line passes through the point $(1, -1)$ that is parallel to the line with the equation $y = 2x - 1$.

**Solution**

**Step 1** Find the slope $m$. The slope of a line parallel to $y = 2x - 1$ is the same as the given line, so the slope is 2.

**Step 2** Find the $y$-intercept $b$ by using $m = 2$ and $(x, y) = (1, -1)$.

$$y = mx + b$$

$-1 = 2(1) + b$ Substitute for $x$, $y$, and $m$. $-3 = b$ Solve for $b$.

Because $m = 2$ and $b = -3$, an equation of the line is $y = 2x - 3$.

**Checkpoint** Complete the following exercises.

1. Write an equation of the line in the graph at the right.
   
   $y = 3x - 5$

2. Write an equation of the line that passes through the point $(-2, 5)$ and is parallel to the line with the equation $y = -2x + 3$.
   
   $y = -2x + 1$
Example 3 Write an equation of a perpendicular line

Write an equation of the line \( j \) passing through the point \((3, 2)\) that is perpendicular to the line \( k \) with the equation \( y = -3x + 1 \).

Solution

Step 1 Find the slope \( m \) of line \( j \). The slope of \( k \) is \( -3 \).

\[
-3 \cdot m = -1
\]

The product of the slopes of perpendicular lines is \( -1 \).

\[
m = \frac{1}{3}
\]

Divide each side by \( -3 \).

Step 2 Find the \( y \)-intercept \( b \) by using \( m = \frac{1}{3} \) and \( (x, y) = (3, 2) \).

\[
y = mx + b
\]

Use slope-intercept form.

\[
2 = \frac{1}{3} (3) + b
\]

Substitute for \( x, y, \) and \( m \).

\[
1 = b
\]

Solve for \( b \).

Because \( m = \frac{1}{3} \) and \( b = 1 \), an equation of line \( j \) is \( y = \frac{1}{3}x + 1 \).

You can check that the lines \( j \) and \( k \) are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.

Checkpoint Complete the following exercise.

3. Write an equation of the line passing through the point \((-8, -2)\) that is perpendicular to the line with the equation \( y = 4x - 3 \).

\[
y = -\frac{1}{4}x - 4
\]
Rent

The graph models the total cost of renting an apartment. Write an equation of the line. Explain the meaning of the slope and the y-intercept of the line.

**Step 1 Find the slope.**

\[ m = \frac{2375 - 1250}{5 - 2} = \frac{1125}{3} = 375 \]

**Step 2 Find the y-intercept. Use a point on the graph.**

\[ y = mx + b \quad \text{Use slope-intercept form.} \]

\[ \frac{1250}{500} = \frac{375 \cdot 2}{b} \quad \text{Substitute.} \]

\[ b = 500 \quad \text{Simplify.} \]

**Step 3 Write the equation.** Because \( m = \frac{375}{3} \) and \( b = 500 \), an equation is \( y = \frac{375x}{1} + 500 \).

The equation \( y = \frac{375x}{3} + 500 \) models the cost. The slope is the **monthly rent**, and the **y-intercept** is the initial cost to rent the apartment.

**Example 5**

**Graph a line with equation in standard form**

Graph \( 2x + 3y = 6 \).

The equation is in standard form, so use the **intercepts**.

**Step 1 Find the intercepts.**

To find the x-intercept, let \( y = 0 \).

\[ 2x + 3(0) = 6 \]

To find the y-intercept, let \( x = 0 \).

\[ 2(0) + 3y = 6 \]

\[ x = \frac{3}{2} \]

\[ y = \frac{2}{3} \]

**Step 2 Graph the line.**

The line intersects the axes at \( (3, 0) \) and \( (0, 2) \). Graph these points, then draw a line through the points.
Subscriptions  You can buy a magazine at a store for $3. You can subscribe yearly to the magazine for a flat fee of $18. After how many magazines is the subscription a better buy?

Solution

Step 1  Model each purchase with an equation.

Cost of yearly subscription:  \( y = 18 \)

Cost of one magazine:  \( y = 3x \), where \( x \) represents the number of magazines

Step 2  Graph each equation.

The point of intersection is \((6, 18)\). Using the graph, you can see that it is cheaper to buy magazines individually if you buy less than 6 magazines per year. If you buy more than 6 magazines per year, it is cheaper to buy a subscription.

Example 6  Solve a real-world problem

4. The equation \( y = 650x + 425 \) models the total cost of joining a health club for \( x \) years. What are the meaning of the slope and \( y \)-intercept of the line?

The slope is the cost per year, $650, and the \( y \)-intercept is the initiation fee, $425.

5. Graph \( y = 3 \) and \( x = 3 \).

6. In Example 6, suppose you can buy the magazine at a different store for $2.50. After how many magazines is the subscription the better buy?

   after 7 magazines
3.6 Prove Theorems About Perpendicular Lines

**Goal**
- Find the distance between a point and a line.

**VOCABULARY**
Distance from a point to a line  The distance from a point to a line is the length of the perpendicular segment from the point to the line.

**THEOREM 3.8**
If two lines intersect to form a linear pair of congruent angles, then the lines are **perpendicular**.

If \( \angle 1 \cong \angle 2 \), then \( g \perp h \).

**THEOREM 3.9**
If two lines are perpendicular, then they intersect to form four **right angles**.

If \( a \perp b \), then \( \angle 1, \angle 2, \angle 3, \) and \( \angle 4 \) are **right angles**.

**Example 1**
**Draw conclusions**

In the diagram at the right, \( \angle 1 \cong \angle 2 \).
What can you conclude about \( a \) and \( b \)?

**Solution**
Lines \( a \) and \( b \) intersect to form a **linear pair of congruent angles**, \( \angle 1 \) and \( \angle 2 \). So, by Theorem 3.8, \( a \perp b \).
**THEOREM 3.10**

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If \( BA \perp BC \), then \( \angle 1 \) and \( \angle 2 \) are complementary.

---

**Example 2**  
*Write a proof*

In the diagram at the right, \( \angle 1 \equiv \angle 2 \).
Prove that \( \angle 3 \) and \( \angle 4 \) are complementary.

**Given** \( \angle 1 \equiv \angle 2 \)

**Prove** \( \angle 3 \) and \( \angle 4 \) are complementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \equiv \angle 2 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( PS \perp PQ )</td>
<td>2. Theorem 3.8</td>
</tr>
<tr>
<td>3. ( \angle 3 ) and ( \angle 4 ) are complementary.</td>
<td>3. Theorem 3.10</td>
</tr>
</tbody>
</table>

**Checkpoint**  
Complete the following exercises.

1. If \( c \perp d \), what do you know about the sum of the measures of \( \angle 3 \) and \( \angle 4 \)?

   *Explain.*

   Because \( c \perp d \), angles 1, 2, 3, and 4 are right angles by Theorem 3.9. So, \( m\angle 3 + m\angle 4 = 180^\circ \).

2. Using the diagram in Example 2, complete the following proof that \( \angle QPS \) and \( \angle 1 \) are right angles.

<table>
<thead>
<tr>
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</thead>
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<tr>
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<tr>
<td>2. ( PS \perp PQ )</td>
<td>2. Theorem 3.8</td>
</tr>
<tr>
<td>3. ( \angle QPS ) and ( \angle 1 ) are right angles.</td>
<td>3. Theorem 3.9</td>
</tr>
</tbody>
</table>
**THEOREM 3.11 PERPENDICULAR TRANSVERSAL THEOREM**

If a transversal is perpendicular to one of two parallel lines, then it is **perpendicular** to the other.

If \( h \parallel k \) and \( j \perp h \), then \( j \perp k \).

**THEOREM 3.12 LINES PERPENDICULAR TO A TRANSVERSAL THEOREM**

In a plane, if two lines are perpendicular to the same line, then they are **parallel** to each other.

If \( m \perp p \) and \( n \perp p \), then \( m \parallel n \).

---

**Example 3**  
**Draw conclusions**

Determine which lines, if any, must be parallel in the diagram. *Explain* your reasoning.

**Solution**

Lines \( r \) and \( s \) are both perpendicular to \( x \), so by Theorem 3.12, \( r \parallel s \). Similarly, lines \( x \) and \( y \) are both perpendicular to \( r \), so \( x \parallel y \). Also, lines \( x \) and \( z \) are both perpendicular to \( s \), so \( x \parallel z \). Finally, because \( y \) and \( z \) are both parallel to \( x \), you know that \( y \parallel z \) by the Transitive Property of Parallel Lines.

**Checkpoint** Use the diagram to complete the following exercises.

3. Is \( c \parallel d \)? *Explain.*

   Yes, because \( c \) and \( d \) are both perpendicular to \( a \), \( c \parallel d \) by Theorem 3.12.

4. Is \( b \perp d \)? *Explain.*

   Yes, because \( b \perp c \), and \( c \parallel d \) as explained in Exercise 3, then \( b \perp d \) by Theorem 3.11.
Example 4  Find the distance between two parallel lines

Railroads  The section of broad gauge railroad track at the right is drawn on a graph where units are measured in inches. What is the distance from \( P \) to segment \( QR \)?

Solution

You need to confirm that segment \( RQ \) is perpendicular to segment \( PQ \).

Using \( Q(71, 34) \) and \( R(91, 55) \), the slope of \( QR \) is

\[
\frac{55 - 34}{91 - 71} = \frac{21}{20}
\]

The segment \( PQ \) has a slope of

\[
\frac{34 - 74}{71 - 29} = \frac{-40}{42} = \frac{-20}{21}
\]

The segment \( PQ \) is perpendicular to segment \( QR \) so \( PQ \) is

\[
d = \sqrt{(29 - 71)^2 + (74 - 34)^2} = 58.
\]

The distance from \( P \) to segment \( QR \) is 58 inches.

Checkpoint  Complete the following exercise.

5. What is the approximate distance from line \( m \) to line \( n \)?

about 3.2 units
Taxicab Geometry

**VOCABULARY**

Taxicab geometry  The non-Euclidean geometry that a taxicab or a pedestrian must obey.

**TAXICAB DISTANCE**

The distance between two points is the sum of the absolute values of the differences in their coordinates.

\[ AB = |x_2 - x_1| + |y_2 - y_1| \]

**Example 1**  *Find a taxicab distance*

Find the taxicab distance from \( A(4, 5) \) to \( B(-3, -4) \). Draw two different shortest paths from \( A \) to \( B \).

**Solution**

\[ AB = |x_2 - x_1| + |y_2 - y_1| \]
\[ = |-3 - 4| + |-4 - 5| \]
\[ = |7| + |9| \]
\[ = 16 \]

The shortest path is **16** blocks.
**Checkpoint** Find the taxicab distance from $A$ to $B$. Draw two different shortest paths from $A$ to $B$.

1. $A(6, 2)$ to $B(-1, -4)$.

![Graph showing A(6, 2) and B(-1, -4)]

**Example 2** *Draw a taxicab circle.*

Draw the taxicab circle with the given radius $r$ and center $C$.

**Solution**

a. $r = 3$, $C(4, 1)$

![Graph showing a taxicab circle with radius 3 centered at (4, 1)]

b. $r = 1$, $C(-4, -6)$

![Graph showing a taxicab circle with radius 1 centered at (-4, -6)]

**Checkpoint** Draw the taxicab circle with the given radius $r$ and center $C$.

2. $r = 2$, $C(-1, -2)$

![Graph showing a taxicab circle with radius 2 centered at (-1, -2)]

3. $r = 4$, $C(-4, 3)$

![Graph showing a taxicab circle with radius 4 centered at (-4, 3)]
## Words to Review

**Give an example of the vocabulary word.**

<table>
<thead>
<tr>
<th>Parallel lines</th>
<th>Skew lines</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Parallel Lines" /></td>
<td><img src="image2" alt="Skew Lines" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parallel planes</th>
<th>Transversal</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Parallel Planes" /></td>
<td><img src="image4" alt="Transversal" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corresponding angles</th>
<th>Alternate interior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Corresponding Angles" /></td>
<td><img src="image6" alt="Alternate Interior Angles" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternate exterior angles</th>
<th>Consecutive interior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7" alt="Alternate Exterior Angles" /></td>
<td><img src="image8" alt="Consecutive Interior Angles" /></td>
</tr>
</tbody>
</table>
Paragraph proof

Given: $m\angle 1 = 50^\circ$
$m\angle 2 = 50^\circ$

Prove: $a \parallel b$

You are given that $m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$ by the definition of congruent angles. So, $a \parallel b$ by the Corresponding Angles Converse.

Slope

The slope of $y = 8x - 7$ is 8.

<table>
<thead>
<tr>
<th>Slope-intercept form</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.</td>
<td>$Ax + By = C$, where $A$ and $B$ are not both zero.</td>
</tr>
</tbody>
</table>

Distance from a point to a line.

The distance from point $A$ to line $b$ is 5.

Taxicab geometry

**Review your notes and Chapter 3 by using the Chapter Review on pages 209–212 of your textbook.**
4.1 Apply Triangle Sum Properties

Goal

- Classify triangles and find measures of their angles.

Your Notes

VOCABULARY

Triangle  A triangle is a polygon with three sides.

Interior angles  When the sides of a polygon are extended, the original angles are the interior angles.

Exterior angles  When the sides of a polygon are extended, the angles that form linear pairs with the interior angles are the exterior angles.

Corollary to a theorem  A corollary to a theorem is a statement that can be proved easily using the theorem.

CLASSIFYING TRIANGLES BY SIDES

<table>
<thead>
<tr>
<th>Scalene Triangle</th>
<th>Isosceles Triangle</th>
<th>Equilateral Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Scalene Triangle" /></td>
<td><img src="image" alt="Isosceles Triangle" /></td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>No congruent sides</td>
<td>At least 2 congruent sides</td>
<td>3 congruent sides</td>
</tr>
</tbody>
</table>

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

CLASSIFYING TRIANGLES BY ANGLES

<table>
<thead>
<tr>
<th>Acute Triangle</th>
<th>Right Triangle</th>
<th>Obtuse Triangle</th>
<th>Equiangular Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Acute Triangle" /></td>
<td><img src="image" alt="Right Triangle" /></td>
<td><img src="image" alt="Obtuse Triangle" /></td>
<td><img src="image" alt="Equiangular Triangle" /></td>
</tr>
<tr>
<td>3 acute angles</td>
<td>1 right angle</td>
<td>1 obtuse angle</td>
<td>3 congruent angles</td>
</tr>
</tbody>
</table>
Example 1  Classify triangles by sides and by angles

Shuffleboard  Classify the triangular shape of the shuffleboard scoring area in the diagram by its sides and by measuring its angles.

Solution

The triangle has a pair of congruent sides, so it is **isosceles**. By measuring, the angles are about 72°, 72°, and 36°. It is an **acute isosceles** triangle.

Checkpoint  Complete the following exercise.

1. Draw an isosceles right triangle and an obtuse scalene triangle.

Sample Drawings:

Example 2  Classify a triangle in a coordinate plane

Classify \( \triangle RST \) by its sides. Then determine if the triangle is a right triangle.

Solution

Step 1  Use the distance formula \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \) to find the side lengths.

\[
RT = \sqrt{(3 - (-3))^2 + (-1 - 3)^2} = \sqrt{52}
\]

\[
RS = \sqrt{(5 - (-3))^2 + (2 - 3)^2} = \sqrt{65}
\]

\[
ST = \sqrt{(3 - 5)^2 + (-1 - 2)^2} = \sqrt{13}
\]

Step 2  Check for right angles. The slope of \( RT \) is \( \frac{-1 - 3}{3 - (-3)} = \frac{-2}{3} \). The slope of \( ST \) is \( \frac{-1 - 2}{3 - 5} = \frac{3}{2} \). The product of the slopes is \( -1 \), so \( RT \perp ST \) and \( \angle RTS \) is a **right** angle. Therefore, \( \triangle RST \) is a **right scalene** triangle.
THEOREM 4.1: TRIANGLE SUM THEOREM
The sum of the measures of the interior angles of a triangle is \(180^\circ\).

\[m\angle A + m\angle B + m\angle C = 180^\circ\]

THEOREM 4.2: EXTERIOR ANGLE THEOREM
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Example 3

**Find angle measure**

Use the diagram at the right to find the measure of \(\angle DCB\).

**Solution**

**Step 1** Write and solve an equation to find the value of \(x\).

\[(3x - 9)^\circ = 73^\circ + x^\circ\]  
Exterior Angle Theorem

\[x = 41^\circ\]  
Solve for \(x\).

**Step 2** Substitute \(41\) for \(x\) in \(3x - 9\) to find \(m\angle DCB\).

\[3x - 9 = 3 \cdot 41 - 9 = 114\]

The measure of \(\angle DCB\) is \(114^\circ\).

COROLLARY TO THE TRIANGLE SUM THEOREM
The acute angles of a right triangle are \(\text{complementary}\).

\[m\angle A + m\angle B = 90^\circ\]
Example 4  Find angle measures from a verbal description

Ramps  The front face of the wheelchair ramp shown forms a right triangle. The measure of one acute angle in the triangle is eight times the measure of the other. Find the measure of each acute angle.

Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be \( x \). Then the measure of the larger acute angle is \( 8x \).

Use the Corollary to the Triangle Sum Theorem to set up and solve an equation.

\[
x^\circ + 8x^\circ = 90^\circ \\
x = 10
\]

So, the measures of the acute angles are \( 10^\circ \) and \( 80^\circ \).

Checkpoint  Complete the following exercises.

2. Triangle \( JKL \) has vertices \( J(-2, -1) \), \( K(1, 3) \), and \( L(5, 0) \). Classify it by its sides. Then determine if it is a right triangle.

   isosceles triangle; right triangle

3. Find the measure of \( \angle 1 \) in the diagram shown.

   \( 56^\circ \)

4. In Example 4, what is the measure of the obtuse angle formed between the ramp and a segment extending from the horizontal leg?

   \( 170^\circ \)
4.2 Apply Congruence and Triangles

Goal • Identify congruent figures.

Your Notes

VOCABULARY
Congruent figures In two congruent figures, all the parts of one figure are congruent to the corresponding parts of the other figure.

Corresponding parts In congruent polygons, the corresponding parts are the corresponding sides and the corresponding angles.

Example 1 Identify congruent parts

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.

Solution
The diagram indicates that $\triangle ABC \cong \triangle FGH$.

Corresponding angles $\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle H$

Corresponding sides $AB \cong FG$, $BC \cong GH$, $CA \cong HF$

Example 2 Use properties of congruent figures

In the diagram, $QRST \cong WXYZ$.

a. Find the value of $x$.
b. Find the value of $y$.

Solution
a. You know $\angle Q \cong \angle W$.

$$m\angle Q = m\angle W$$

$$65^\circ = (5x + 5)^\circ$$

$$60 = 5x$$

$$12 = x$$

b. You know $QR \cong WX$.

$$QR = WX$$

$$6 = y - x$$

$$6 = y - 12$$

$$18 = y$$
1. Identify all pairs of congruent corresponding parts.

Corresponding angles: \( \angle F \cong \angle S, \angle G \cong \angle T, \angle H \cong \angle U, \angle J \cong \angle V \)

Corresponding sides: \( FG \cong ST, \; GH \cong TU, \; HJ \cong UV, \; JF \cong VS \)

2. Find the value of \( x \) and find \( m\angle G \).

\( x = 55; \; m\angle G = 103^\circ \)

Example 3  **Show that figures are congruent**

Maps  If you cut the rectangular map in half along \( PR \), will the sections of the map be the same size and shape? *Explain.*

**Solution**

From the diagram, \( \angle S \cong \angle Q \) because all right angles are congruent. Also, by the **Lines Perpendicular to a Transversal Theorem**, \( PQ \parallel RS \). Then \( \angle 1 \cong \angle 4 \) and \( \angle 2 \cong \angle 3 \) by the **Alternate Interior Angles Theorem**. So, all pairs of corresponding angles are **congruent**.

The diagram shows \( PQ \cong RS \) and \( QR \cong SP \). By the **Reflexive Property**, \( PR \cong RP \). All corresponding parts are **congruent**, so \( \triangle PQR \cong \triangle RSP \).

*Yes*, the two sections will be the same **size** and **shape**.
THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

Example 4 Use the Third Angles Theorem

Find \( m\angle V \).

Solution

\( \angle SUT \cong \angle VUW \) by the Vertical Angles Theorem. The diagram shows that \( \angle STU \cong \angle VWU \), so by the Third Angles Theorem, \( \angle S \cong \angle V \). By the Triangle Sum Theorem, \( m\angle S = 180^\circ - 66^\circ - 44^\circ = 70^\circ \). So, \( m\angle S = m\angle V = 70^\circ \) by the definition of congruent angles.

Example 5 Prove that triangles are congruent

Write a proof.

Given \( FH \cong JH, FG \cong JG, \angle FHG \cong \angle JHG, \angle FGH \cong \angle JGH \)

Prove \( \triangle FGH \cong \triangle JGH \)

Plan for Proof
a. Use the Reflexive Property to show \( HG \cong HG \).
b. Use the Third Angles Theorem to show \( \angle F \cong \angle J \).

Plan in Action

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( FH \cong JH, FG \cong JG )</td>
<td>1. Given</td>
</tr>
<tr>
<td>a. 2. ( HG \cong HG )</td>
<td>2. Reflexive Property of Congruence</td>
</tr>
<tr>
<td>3. ( \angle FHG \cong \angle JHG, \angle FGH \cong \angle JGH )</td>
<td>3. Given</td>
</tr>
<tr>
<td>b. 4. ( \angle F \cong \angle J )</td>
<td>4. Third Angles Theorem</td>
</tr>
<tr>
<td>5. ( \triangle FGH \cong \triangle JGH )</td>
<td>5. Definition of ( \cong ) ( \triangle s )</td>
</tr>
</tbody>
</table>
**THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES**

**Reflexive Property of Congruent Triangles**
For any triangle \( \triangle ABC \), \( \triangle ABC \cong \triangle ABC \).

**Symmetric Property of Congruent Triangles**
If \( \triangle ABC \cong \triangle DEF \), then \( \triangle DEF \cong \triangle ABC \).

**Transitive Property of Congruent Triangles**
If \( \triangle ABC \cong \triangle DEF \) and \( \triangle DEF \cong \triangle JKL \), then \( \triangle ABC \cong \triangle JKL \).

**Checkpoint** Complete the following exercises.

3. In the diagram at the right, \( E \) is the midpoint of \( AC \) and \( BD \).
   Show that \( \triangle ABE \cong \triangle CDE \).

   From the diagram, \( AB \cong CD \). Point \( E \) is the midpoint of \( AC \) and \( BD \), so \( AE \cong CE \) and \( BE \cong DE \) by the definition of midpoint. So all pairs of corresponding sides are congruent.

   The diagram shows \( AB \parallel CD \), so \( \angle A \cong \angle C \) and \( \angle B \cong \angle D \) by the Alternate Interior Angles Theorem. Also, \( \angle AEB \cong \angle CED \) by the Vertical Angles Theorem. All corresponding parts are congruent, so \( \triangle ABE \cong \triangle CDE \).

4. In the diagram, what is the measure of \( \angle D \)?
   \( 62^\circ \)

5. By the definition of congruence, what additional information is needed to know that \( \triangle ABE \cong \triangle DCE \) in Exercise 4?
   You must know that \( AB \cong DC \) and \( BE \cong CE \) to conclude that \( \triangle ABE \cong \triangle DCE \). The remaining information can be inferred from the graph.
4.3 Prove Triangles Congruent by SSS

**Goal** • Use side lengths to prove triangles are congruent.

**POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE**

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{AB} \cong \overline{RS}$,  
Side $\overline{BC} \cong \overline{ST}$, and  
Side $\overline{CA} \cong \overline{TR}$,  
then $\triangle ABC \cong \triangle RST$.

**Example 1** Use the SSS Congruence Postulate

Write a proof.

Given $\overline{FJ} \cong \overline{HJ}$,  
G is the midpoint of $\overline{FH}$.

Prove $\triangle FGJ \cong \triangle HGJ$

Proof It is given that $\overline{FJ} \cong \overline{HJ}$. Point G is the midpoint of $\overline{FH}$, so $\overline{FG} \cong \overline{HG}$. By the Reflexive Property, $\overline{GJ} \cong \overline{JG}$. So, by the SSS Congruence Postulate, $\triangle FGJ \cong \triangle HGJ$.

**Checkpoint** Decide whether the congruence statement is true. Explain your reasoning.

1. $\triangle JKL \cong \triangle MKL$  
   True; all corresponding sides are congruent.

2. $\triangle RST \cong \triangle VW$  
   False; $\overline{RS} \not\cong \overline{TV}$
Determine whether \( \triangle PQR \) is congruent to the other triangles shown at the right.

**Solution**

By counting, \( PQ = 3 \) and \( QR = 5 \). Use the distance formula to find \( PR \).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
PR = \sqrt{(2 - (-3))^2 + (2 - 5)^2} = \sqrt{34}
\]

By the SSS Congruence Postulate, any triangle with side lengths \( 3 \), \( 5 \), and \( \sqrt{34} \) will be congruent to \( \triangle PQR \). The distance from \( R \) to \( S \) is \( 3 \). The distance from \( R \) to \( T \) is \( 5 \). The distance from \( S \) to \( T \) is \( \sqrt{(2 - 5)^2 + (-3 - 2)^2} = \sqrt{34} \). So, \( \triangle PQR \cong \triangle SRT \).

The distance from \( W \) to \( V \) is

\[
\sqrt{(-3 - 0)^2 + (-2 - (-3))^2} = \sqrt{10}
\]

No side of \( \triangle PQR \) has a length of \( \sqrt{10} \), so \( \triangle PQR \not\cong \triangle VWR \).

**Checkpoint** Complete the following exercise.

3. \( \triangle DFG \) has vertices \( D(-2, 4) \), \( F(4, 4) \), and \( G(-2, 2) \). \( \triangle LMN \) has vertices \( L(-3, -3) \), \( M(-3, 3) \), and \( N(-1, -3) \). Graph the triangles in the same coordinate plane and show that they are congruent.

\[
DG = LN = 2, \ DF = LM = 6, \text{ and } \ FG = MN = \sqrt{40}, \text{ so } \triangle DFG \cong \triangle LMN \text{ by the SSS Congruence Postulate.} 
\]
Example 3  Solve a real-world problem

Stability  Explain why the table with the diagonal legs is stable, while the one without the diagonal legs can collapse.

Solution

The table with the diagonal legs forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the table is stable. The table without the diagonal legs is not stable because there are many possible quadrilaterals with the given side lengths.

Checkpoint  Determine whether the figure is stable. Explain your reasoning.

4. Yes, the figure is stable. By the SSS Congruence Postulate, the triangles formed cannot change shape, so it is stable.

5. No, the figure is not stable. There are many possible quadrilaterals with the given side lengths.
**Lesson 4.4**

**Goal**
- Use sides and angles to prove congruence.

**VOCABULARY**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leg of a right triangle</td>
<td>In a right triangle, a side adjacent to the right angle is called a leg.</td>
</tr>
<tr>
<td>Hypotenuse</td>
<td>In a right triangle, the side opposite the right angle is called the hypotenuse.</td>
</tr>
</tbody>
</table>

**POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE**

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong \overline{UV}$, Angle $\angle R \cong \angle U$, and Side $\overline{RT} \cong \overline{UW}$, then $\triangle RST \cong \triangle UVW$.

**Example 1**

**Use the SAS Congruence Postulate**

Write a proof.

**Given** $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$

**Prove** $\triangle JKN \cong \triangle LMN$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 2$</td>
<td>2. Vertical Angles Theorem</td>
</tr>
<tr>
<td>3. $\triangle JKN \cong \triangle LMN$</td>
<td>3. SAS Congruence Postulate</td>
</tr>
</tbody>
</table>

**Example 2**

**Use the HL Congruence Theorem**

Write a proof.

**Given** $\overline{JL} \perp \overline{KM}$, $\overline{LN} \perp \overline{LM}$

**Prove** $\triangle JLN \cong \triangle KLN$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Vertical Angles Theorem</td>
</tr>
<tr>
<td>2. $\overline{JL} \cong \overline{KL}$, $\overline{LN} \cong \overline{LN}$</td>
<td>2. Hypotenuse-Leg (HL) Congruence Theorem</td>
</tr>
<tr>
<td>3. $\triangle JLN \cong \triangle KLN$</td>
<td>3. HL Congruence Theorem</td>
</tr>
</tbody>
</table>
Example 2  **Use SAS and properties of shapes**

In the diagram, \(ABCD\) is a rectangle. What can you conclude about \(\triangle ABC\) and \(\triangle CDA\)?

**Solution**

By the Right Angles Congruence Theorem, \(\angle B \cong \angle D\). Opposite sides of a rectangle are congruent, so \(AB \cong CD\) and \(BC \cong DA\).

\(\triangle ABC\) and \(\triangle CDA\) are congruent by the SAS Congruence Postulate.

**Checkpoint** In the diagram, \(AB\), \(CD\), and \(EF\) pass through the center \(M\) of the circle. Also, \(\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4\).

1. Prove that \(\triangle DMY \cong \triangle BMY\).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\angle 3 \cong \angle 4)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (DM \cong BM)</td>
<td>2. Definition of a circle</td>
</tr>
<tr>
<td>3. (MY \cong MY)</td>
<td>3. Reflexive Property of Congruence</td>
</tr>
<tr>
<td>4. (\triangle DMY \cong \triangle BMY)</td>
<td>4. SAS Congruence Postulate</td>
</tr>
</tbody>
</table>

2. What can you conclude about \(AC\) and \(BD\)?

Because they are vertical angles, \(\angle AMC \cong \angle BMD\). All points on a circle are the same distance from the center, so \(AM = BM = CM = DM\). By the SAS Congruence Postulate, \(\triangle AMC \cong \triangle BMD\). Corresponding parts of congruent triangles are congruent, so you know \(AC \cong BD\).
THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are congruent.

Example 3  Use the Hypotenuse-Leg Theorem

Write a proof.

Given

\[ \overline{AC} \cong \overline{EC}, \]
\[ \overline{AB} \perp \overline{BD}, \]
\[ \overline{ED} \perp \overline{BD}, \]
\[ \overline{AC} \text{ is a bisector of } \overline{BD}. \]

Prove

\[ \triangle ABC \cong \triangle EDC \]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>H 1. ( \overline{AC} \cong \overline{EC} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{AB} \perp \overline{BD} ), ( \overline{ED} \perp \overline{BD} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle B ) and ( \angle D ) are right angles.</td>
<td>3. Definition of ( \perp ) lines</td>
</tr>
<tr>
<td>4. ( \triangle ABC ) and ( \triangle EDC ) are right triangles.</td>
<td>4. Definition of a right triangle</td>
</tr>
<tr>
<td>5. ( \overline{AC} ) is a bisector of ( \overline{BD} ).</td>
<td>5. Given</td>
</tr>
<tr>
<td>L 6. ( \overline{BC} \cong \overline{DC} )</td>
<td>6. Definition of segment bisector</td>
</tr>
<tr>
<td>7. ( \triangle ABC \cong \triangle EDC )</td>
<td>7. HL Congruence Theorem</td>
</tr>
</tbody>
</table>
3. Explain why a diagonal of a rectangle forms a pair of congruent triangles.

   A diagonal of a rectangle will be the hypotenuse of each triangle formed. Because the hypotenuse is congruent to itself, and because opposite sides of a rectangle are congruent, you can use the HL Congruence Theorem to conclude the triangles are congruent.

4. In Example 4, suppose it is given that $ABCF$ and $EDCF$ are squares. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$? Explain.

   It is given that $ABCF$ and $EDCF$ are squares, so $\angle B$ and $\angle D$ are right angles, $AB \cong DE$, and $BC \cong DC$. You can use the SAS Congruence Postulate to conclude that $\triangle ABC \cong \triangle EDC$. 
Lesson 4.5
Prove Triangles Congruent
by ASA and AAS

Goal • Use two more methods to prove congruences.

VOCABULARY

Flow proof  A flow proof uses arrows to show the flow of a logical argument.

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D \),
\( \overline{AC} \cong \overline{DF} \), and
\( \angle C \cong \angle F \),
then \( \triangle ABC \cong \triangle DEF \).

THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D \),
\( \angle C \cong \angle F \), and
\( \overline{BC} \cong \overline{EF} \),
then \( \triangle ABC \cong \triangle DEF \).
Example 1  Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. [Diagram]

b. [Diagram]

c. [Diagram]

Solution

a. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

b. Two pairs of angles and a non-included pair of sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

c. The vertical angles are congruent, so two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

Checkpoint  Can $\triangle STW$ and $\triangle VWT$ be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

1. [Diagram]

   Yes; AAS Congruence Theorem

2. [Diagram]

   Yes; ASA Congruence Postulate
Example 2  Write a flow proof

In the diagram, $\angle 1 \cong \angle 4$ and $\overline{CF}$ bisects $\angle ACE$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

Solution

Given $\angle 1 \cong \angle 4$, $\overline{CF}$ bisects $\angle ACE$.

Prove $\triangle CBF \cong \triangle CDF$

\[
\begin{align*}
\angle 1 \cong \angle 4 & \quad \text{Given} \\
\angle 1 \text{ and } \angle 2 \text{ are supplements} & \quad \text{Linear Pair Postulate} \\
\angle 3 \text{ and } \angle 4 \text{ are supplements} & \quad \text{Given} \\
\angle 2 \cong \angle 3 & \quad \text{Congruent Supps. Thm.} \\
\overline{CF} \cong \overline{CF} & \quad \text{Reflexive Prop.} \\
\angle ACF \cong \angle ECF & \quad \text{Def. of $\angle$ bisector} \\
\angle CBF \cong \angle CDF & \quad \text{AAS Congruence Theorem}
\end{align*}
\]

Checkpoint  Complete the following exercise.

3. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

\[
\begin{align*}
\overline{CF} \text{ bisects } \angle ACE. & \quad \text{Given} \\
\angle ACF \cong \angle ECF & \quad \text{Def. of $\angle$ bisector} \\
\overline{CF} \text{ bisects } \angle BFD. & \quad \text{Given} \\
\overline{BCF} \cong \angle DFC & \quad \text{Def. of $\angle$ bisector} \\
\angle CBF \cong \angle CDF & \quad \text{ASA Congruence Postulate}
\end{align*}
\]
Games You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is 72° and the angle from your friend to the flag is 53°. Is there enough information to locate the flag?

Solution

The locations of you, your friend, and the flag form a triangle. The measures of two angles and an included side of the triangle are known.

By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the flag location is given by the third vertex.

Checkpoint Complete the following exercise.

4. Theater Two actors are standing apart from each other on the edge of a stage. Spotlights are located and pointed as shown in the diagram. Can one of the actors move to another location on the stage without changing any of the angles of the triangle, without changing the distance to the other actor, and without requiring a spotlight to move?

No. The measures of two angles and a nonincluded side of the triangle are known. By the AAS Congruence Theorem, all triangles with these measures are congruent. To create a different congruent triangle, one of the actors would have to move to a location that is off the stage, and the spotlight following that actor would have to move.
Explain how you can use the given information to prove that the triangles are congruent.

Given \( \angle 1 \cong \angle 2, \overline{AB} \cong \overline{DE} \)

Prove \( \overline{DC} \cong \overline{AC} \)

Solution

If you can show that \( \triangle ABC \cong \triangle DEC \), you will know that \( \overline{DC} \cong \overline{AC} \). First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, \( \angle ABC \) and \( \angle DEC \) are supplementary to congruent angles, so \( \angle ABC \cong \angle DEC \). Also, \( \angle ACB \cong \angle DCE \).

Mark given information. Add deduced information.

Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, \( \triangle ABC \cong \triangle DEC \). Because corresponding parts of congruent triangles are congruent, \( \overline{DC} \cong \overline{AC} \).
Boats

Use the following method to find the distance between two docked boats, from point A to point B.

- Place a marker at D so that \( \overline{AB} \perp \overline{BD} \).
- Find C, the midpoint of \( \overline{BD} \).
- Locate the point E so that \( \overline{BD} \perp \overline{DE} \) and A, C, and E are collinear.
- Explain how this plan allows you to find the distance.

**Solution**

Because \( \overline{AB} \perp \overline{BD} \) and \( \overline{BD} \perp \overline{DE} \), \( \angle B \) and \( \angle D \) are congruent right angles. Because C is the midpoint of \( \overline{BD} \), \( \overline{BC} \cong \overline{DC} \). The vertical angles \( \angle ACB \) and \( \angle ECD \) are congruent. So, \( \triangle CBA \cong \triangle CDE \) by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, \( \overline{BA} = \overline{DE} \). So, you can find the distance \( \overline{AB} \) between the boats by measuring \( \overline{DE} \).

**Checkpoint** Complete the following exercises.

1. **Explain** how you can prove that \( \overline{PR} \cong \overline{QS} \).

   Use the AAS Congruence Theorem to show \( \triangle PTS \cong \triangle QTR \). Because corresponding parts of congruent triangles are congruent, \( \overline{PT} \cong \overline{QT} \). Then \( \overline{PR} \cong \overline{QS} \) because \( \overline{ST} \cong \overline{RT} \).

2. **In Example 2**, does it matter how far away from point B you place a marker at point D? **Explain**.

   Point D should be placed far enough away from point B so that it is on land. This allows \( \overline{DE} \) to be easily measured. However, the method will work regardless of how far D is from B.
Use the given information to write a plan for proof.

Given \( \triangle 1 \cong \triangle 2, \angle 3 \cong \angle 4 \)

Prove \( \triangle ABD \cong \triangle ACD \)

**Solution**

In \( \triangle ABD \) and \( \triangle ACD \), you know that \( \angle 1 \cong \angle 2 \) and \( \overline{AD} \cong \overline{AD} \). If you can show that \( \overline{BD} \cong \overline{CD} \), you can use the **SAS Congruence Postulate**.

To prove that \( \overline{BD} \cong \overline{CD} \), you can first prove that \( \triangle BED \cong \triangle CED \). You are given \( \angle 1 \cong \angle 2 \) and \( \angle 3 \cong \angle 4 \). \( \overline{ED} \cong \overline{ED} \) by the Reflexive Property and \( \angle BDE \cong \angle CDE \) by the Congruent Supplements Theorem. You can use the **AAS Congruence Theorem** to prove that \( \triangle BED \cong \triangle CED \).

**Plan for Proof** Use the **AAS Congruence Theorem** to prove that \( \triangle BED \cong \triangle CED \). Then state that \( \overline{BD} \cong \overline{CD} \). Use the **SAS Congruence Postulate** to prove that \( \triangle ABD \cong \triangle ACD \).

**Checkpoint** Use the given information to write a plan for proof.

3. Given \( \overline{GH} \cong \overline{KJ}, \overline{FG} \cong \overline{LK} \), \( \angle FJG \) and \( \angle LHK \) are rt. \( \triangle \).

Prove \( \triangle FJK \cong \triangle LHG \)

Plan for Proof: Use the HL Congruence Theorem to prove that \( \triangle FJG \cong \triangle LHK \). Then state that \( \overline{FJ} \cong \overline{LH} \). Then show that \( \angle FJK \cong \angle LHG \) and use the SAS Congruence Postulate to prove that \( \triangle FJK \cong \triangle LHG \).
Example 4  Justify a construction

Write a proof to justify that the construction for copying an obtuse angle is valid.

Solution

Add \(BC\) and \(EF\) to the diagram. In the construction, \(AB\), \(DE\), \(AC\), and \(DF\) are determined by the same compass setting, as are \(BC\) and \(EF\). So, you can assume the following as given statements.

Given \(AB \cong DE\), \(AC \cong DF\), \(BC \cong EF\)

Prove \(\angle D \cong \angle A\)

Plan for Proof

Show that \(\triangle CAB \cong \triangle FDE\), so you can conclude that the corresponding parts \(\angle D\) and \(\angle A\) are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Plan for Action</td>
<td></td>
</tr>
<tr>
<td>1. (AB \cong DE), (AC \cong DF), (BC \cong EF)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (\triangle CAB \cong \triangle FDE)</td>
<td>2. SSS Congruence Postulate</td>
</tr>
<tr>
<td>3. (\angle D \cong \angle A)</td>
<td>3. Corresp. parts of (\cong) triangles are (\cong).</td>
</tr>
</tbody>
</table>

Checkpoint  Complete the following exercise.

4. Write a paragraph proof to verify that the construction for bisecting a right angle is valid.

You know that \(AC \cong AB\) and \(BD \cong CD\) because they are determined by the same compass settings. Also, \(AD \cong AD\) by the Reflexive Property. So, by the SSS Congruence Postulate, \(\triangle CAD \cong \triangle BAD\). Thus, \(\angle CAD \cong \angle BAD\) because corresponding parts of congruent triangles are congruent.
4.7 Use Isosceles and Equilateral Triangles

**Goal**
- Use theorems about isosceles and equilateral triangles.

**Your Notes**

<table>
<thead>
<tr>
<th>VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legs</td>
</tr>
<tr>
<td>Vertex angle</td>
</tr>
<tr>
<td>Base</td>
</tr>
<tr>
<td>Base angles</td>
</tr>
</tbody>
</table>

**THEOREM 4.7: BASE ANGLES THEOREM**

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

**THEOREM 4.8: CONVERSE OF BASE ANGLES THEOREM**

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

**Example 1**  
*Apply the Base Angles Theorem*

In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.

**Solution**

$\overline{FH} \cong \overline{GH}$, so by the Base Angles Theorem, $\angle F \cong \angle G$. 
Find the values of $x$ and $y$ in the diagram.

**Solution**

**Step 1** Find the value of $x$. Because $\triangle JKL$ is equiangular, it is also equilateral and $KL \equiv JL$. Therefore, $x = 8$.

**Step 2** Find the value of $y$. Because $\angle JML \equiv \angle LJM$, $LM \equiv LJ$, and $\triangle LMJ$ is isosceles. You know that $LJ = 8$.

$LM = LJ$ \hspace{1cm} \text{Definition of congruent segments}

$2y = 8$ \hspace{1cm} \text{Substitute 2y for LM and 8 for LJ.}

$y = 4$ \hspace{1cm} \text{Divide each side by 2.}
Lesson 4.7  •  Geometry Notetaking Guide

Example 4  Solve a multi-step problem

Quilting  The pattern at the right is present in a quilt.

a.  Explain why \( \triangle ADC \) is equilateral.

b.  Show that \( \triangle CBA \cong \triangle ADC \).

Solution

a.  By the Base Angles Theorem, \( \angle DAC \cong \angle DCA \). So, \( \triangle ADC \) is equiangular. By the Corollary to the Converse of Base Angles Theorem, \( \triangle ADC \) is equilateral.

b.  By the Base Angles Theorem, \( \angle ABC \cong \angle ACB \). So, \( \triangle CBA \cong \triangle ADC \) by the AAS Congruence Theorem.

Checkpoint  Complete the following exercises.

1. Copy and complete the statement:
   If \( FH \cong FJ \), then \( \angle ? \cong \angle ? \).
   \( H, J \)

2. Copy and complete the statement:
   If \( \triangle FGK \) is equiangular and \( FG = 15 \), then \( GK = ? \).
   \( 15 \)

3. Use parts (a) and (b) in Example 4 to show that \( m\angle BAD = 120^\circ \).
   \( \triangle DCA \) is equiangular. So, \( m\angle ADC = m\angle DCA = m\angle CAD \).
   \[ 3(m\angle CAD) = 180^\circ \quad \text{Triangle Sum Theorem} \]
   \[ m\angle CAD = 60^\circ \quad \text{Divide each side by 3.} \]
   Because \( \triangle DCA \) is equiangular and \( \triangle CBA \cong \triangle ADC \), you know that \( m\angle BAC = 60^\circ \).
   \[
   m\angle BAD = m\angle BAC + m\angle CAD
   = 60^\circ + 60^\circ
   = 120^\circ
   \]
4.8 Congruence Transformations and Coordinate Geometry

Goal
- Create an image congruent to a given triangle.

Your Notes

VOCABULARY

Transformation
A transformation is an operation that moves or changes a geometric figure in some way to produce a new figure.

Image
The new figure produced by a transformation is the image.

Translation
A translation moves every point of a figure the same distance in the same direction.

Reflection
A reflection uses a line of reflection to create a mirror image of the original figure.

Rotation
A rotation turns a figure about a fixed point, called the center of rotation.

Congruence Transformation
A congruence transformation changes the position of a figure without changing its size or shape.

Example 1
Identify transformations

Name the type of transformation demonstrated in each picture.

a. Rotation about a point

b. Translation in a straight path

c. Reflection in a vertical line

Example 1
Identify transformations

Name the type of transformation demonstrated in each picture.

a. Rotation about a point

b. Translation in a straight path

c. Reflection in a vertical line
COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

\[(x, y) \rightarrow (x + a, y + b)\]

which shows that each point \((x, y)\)
of the unshaded figure is translated horizontally \(a\) units and vertically \(b\) units.

Example 2

Translate a figure in the coordinate plane

Figure \(ABCD\) has the vertices \(A(1, 2), B(3, 3), C(4, -1),\) and \(D(1, -2)\). Sketch \(ABCD\) and its image after the translation \((x, y) \rightarrow (x - 4, y + 2)\).

Solution

First draw \(ABCD\). Find the translation of each vertex by subtracting 4 from its \(x\)-coordinate and adding 2 to its \(y\)-coordinate. Then draw \(ABCD\) and its image.

\[
\begin{align*}
(x, y) & \rightarrow (x - 4, y + 2) \\
A(1, 2) & \rightarrow (-3, 4) \\
B(3, 3) & \rightarrow (-1, 5) \\
C(4, -1) & \rightarrow (0, 1) \\
D(1, -2) & \rightarrow (-3, 0)
\end{align*}
\]

COORDINATE NOTATION FOR A REFLECTION

Reflection in the \(x\)-axis

\[
(x, y) \rightarrow (x, -y)
\]

Multiply \(y\)-coordinate by \(-1\).

Reflection in the \(y\)-axis

\[
(-x, y) \rightarrow (-x, y)
\]

Multiply \(x\)-coordinate by \(-1\).
Shapes You are cutting figures out of paper. Use a reflection in the $x$-axis to draw the other half of the figure.

Solution
Multiply the $y$-coordinate of each vertex by $-1$ to find the corresponding vertex in the image. Then draw the image.

$$(x, y) \rightarrow (x, -y)$$

$$(-1, 0) \rightarrow (-1, 0)$$

$$(1, 1) \rightarrow (1, -1)$$

$$(2, 4) \rightarrow (2, -4)$$

$$(3, 1) \rightarrow (3, -1)$$

$$(7, 0) \rightarrow (7, 0)$$

You can check your results by looking to see if each original point and its image are the same distance from the $x$-axis.

Checkpoint Complete the following exercises.

1. Name the type of transformation shown.
   Translation

2. Figure $FGHJ$ has the vertices $F(0, 2)$, $G(2, 3)$, $H(3, 3)$, and $J(0, -2)$. Sketch $FGHJ$ and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the $y$-axis.
Example 4  Identify a rotation

Graph \( JK \) and \( LM \). Tell whether \( LM \) is a rotation of \( JK \) about the origin. If so, give the angle and direction of rotation.

a. \( J(3, 1), K(1, 4), L(−1, 3), M(−4, 1) \)

b. \( J(−2, 1), K(−1, 5), L(1, 1), M(2, 5) \)

Solution

\[ \triangle JOL \] \[ \triangle KOM \]

\[ m∠JOL = m∠KOM \]

\[ 90° \] counterclockwise rotation

\[ m∠JOL > m∠KOM \]

not a rotation

Checkpoint  Graph \( RS \) and \( TV \). Tell whether \( TV \) is a rotation of \( RS \) about the origin. If so, give the angle of rotation.

3. \( R(−3, −2), S(−3, 2), T(−1, 2), V(3, 2) \)

4. \( R(−1, 1), S(−4, 2), T(1, −1), V(4, −2) \)

\[ m∠ROT < m∠SOV \]

not a rotation

\[ m∠ROT = m∠SOV \]

\[ 180° \] rotation
Example 5  
Verify congruence

The vertices of \( \triangle PQR \) are \( P(2, 2) \), \( Q(3, 4) \), and \( R(5, 2) \). The notation \( (x, y) \rightarrow (x + 1, y - 6) \) describes the translation of \( \triangle PQR \) to \( \triangle XYZ \). Show that \( \triangle PQR \cong \triangle XYZ \) to verify that the translation is a congruence transformation.

Solution

S  You can see that \( PR = XZ = 3 \), so \( PR \cong XZ \).

A  Using the slopes, \( \overline{PQ} \parallel \overline{XY} \) and \( \overline{QR} \parallel \overline{YZ} \). If you extend \( PQ \) and \( XZ \) to form \( \triangle V \), the Corresponding Angles Postulate gives you \( \angle QPR \cong \angle V \) and \( \angle V \cong \angle YXZ \). Then, \( \angle QPR \cong \angle YXZ \) by the Transitive Property of Congruence.

S  Using the distance formula, \( PQ = XY = \sqrt{5} \) so \( PQ \cong XY \). So, \( \triangle PQR \cong \triangle XYZ \) by the SAS Congruence Postulate.

Because \( \triangle PQR \cong \triangle XYZ \), the translation is a congruence transformation.

Checkpoint  Complete the following exercise.

5. Show that \( \triangle ABC \cong \triangle EDC \) to verify that the transformation is a congruence transformation.

Homework

You can see that \( AB = ED = 4 \) and \( BC = DC = 3 \). Also, \( \angle B \) and \( \angle D \) are congruent right angles. So, \( \triangle ABC \cong \triangle EDC \) by the SAS Congruence Postulate.
## Words to Review

**Give an example of the vocabulary word.**

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Scalene triangle</th>
</tr>
</thead>
<tbody>
<tr>
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<td><img src="scalene_triangle.png" alt="Scalene triangle" /></td>
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<table>
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<tr>
<th>Isosceles triangle</th>
<th>Equilateral triangle</th>
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<table>
<thead>
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<th>Right triangle</th>
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<table>
<thead>
<tr>
<th>Obtuse triangle</th>
<th>Equiangular triangle</th>
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<tr>
<td><img src="obtuse_triangle.png" alt="Obtuse triangle" /></td>
<td><img src="equiangular_triangle.png" alt="Equiangular triangle" /></td>
</tr>
<tr>
<td>Interior angles</td>
<td>Exterior angles</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>( \angle 1, \angle 2, ) and ( \angle 3 ) are interior angles.</td>
<td>( \angle 4, \angle 5, ) and ( \angle 6 ) are exterior angles.</td>
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</table>

<table>
<thead>
<tr>
<th>Corollary to a theorem</th>
<th>Congruent figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>A corollary to a theorem is a statement that can be proved easily using the theorem.</td>
<td>( \triangle ABC \cong \triangle DEF )</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Corresponding parts</th>
<th>Leg of a right triangle, hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( \triangle ABC \cong \triangle DEF ), then ( \angle A ) and ( \angle D ) are corresponding parts.</td>
<td>Leg of a right triangle, hypotenuse</td>
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<table>
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<th>Legs, base of an isosceles triangle</th>
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<tr>
<td>Statement 1</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Reason 1</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Statement 2</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Reason 2</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Statement 3</td>
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<tr>
<td>Reason 3</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Statement 4</td>
<td>legs, base of an isosceles triangle</td>
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<tr>
<td>Reason 4</td>
<td>legs, base of an isosceles triangle</td>
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<td><strong>Vertex angle, base angles of an isosceles triangle</strong></td>
<td><strong>Transformation</strong></td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
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<td>[vertex angle][base angles]</td>
<td><a href="translation">image</a></td>
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<td>[image]</td>
<td>[translation]</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Reflection</strong></th>
<th><strong>Rotation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>[reflection](y axis)</td>
<td><a href="P">rotation</a></td>
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</table>

**Congruence transformation**

Translations, reflections, and rotations are three types of congruence transformations. Because they change the position of a figure without changing its size or shape.

**Review your notes and Chapter 4 by using the Chapter Review on pages 295–298 of your textbook.**
5.1 Midsegment Theorem and Coordinate Proof

**Goal**
- Use properties of midsegments and write coordinate proofs.

**VOCABULARY**

- **Midsegment of a triangle**: A midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle.

- **Coordinate proof**: A coordinate proof involves placing geometric figures in a coordinate plane.

**THEOREM 5.1: MIDSEGMENT THEOREM**
The segment connecting the midpoints of two sides of a triangle is **parallel** to the third side and is **half** as long as that side.

\[ DE \parallel AC \text{ and } DE = \frac{1}{2} AC \]

**Example 1**
**Use the Midsegment Theorem to find lengths**

**Windows** A large triangular window is segmented as shown. In the diagram, \( DF \) and \( EF \) are midsegments of \( \triangle ABC \). Find \( DF \) and \( AB \).

**Solution**

\[
DF = \frac{1}{2} \cdot BC = \frac{1}{2} (90 \text{ in.}) = 45 \text{ in.}
\]

\[
AB = 2 \cdot FE = 2 (45 \text{ in.}) = 90 \text{ in.}
\]

**Checkpoint** Complete the following exercise.

1. In Example 1, consider \( \triangle ADF \). What is the length of the midsegment opposite \( DF \)?

22.5 in.
Example 2  Use the Midsegment Theorem

In the diagram at the right, \( QS = SP \) and \( PT = TR \). Show that \( QR \parallel ST \).

Solution

Because \( QS = SP \) and \( PT = TR \), \( S \) is the midpoint of \( QP \) and \( T \) is the midpoint of \( PR \) by definition. Then \( ST \) is a midsegment of \( \triangle PQR \) by definition and \( QR \parallel ST \) by the Midsegment Theorem.

☐ Checkpoint  Complete the following exercise.

2. In Example 2, if \( V \) is the midpoint of \( QR \), what do you know about \( SV \)?

\( SV \) is a midsegment of \( \triangle PQR \) and \( SV \parallel PR \).

Example 3  Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. a square  b. an acute triangle

Solution

It is easy to find lengths of horizontal and vertical segments and distances from \( (0, 0) \), so place one vertex at the origin and one or more sides on an axis.

a. Let \( s \) represent the side length.

b. You need to use three different variables.
In Example 3 part (a), find the length and midpoint of a diagonal of the square.

Solution

Draw a diagonal and its midpoint. Assign letters to the points.

Use the distance formula to find $BD$.

$$BD = \sqrt{(s - 0)^2 + (s - 0)^2}$$

$$= \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2}$$

Use the midpoint formula to find the midpoint $M$.

$$M\left(\frac{s + 0}{2}, \frac{s + 0}{2}\right) = M\left(\frac{s}{2}, \frac{s}{2}\right)$$

**Checkpoint** Complete the following exercises.

3. Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

*Sample answer:*

![Sample answer](image)

4. In Example 4, find the length and midpoint of diagonal $\overline{AC}$. What do you notice? *Explain* why this is true for all squares.

length: $s\sqrt{2}$; midpoint: $M\left(\frac{s}{2}, \frac{s}{2}\right)$; The lengths of the diagonals are the same, and the midpoints of the diagonals are the same. This is true for all squares because the coordinates are based only on the definition of a square.
5.2 Use Perpendicular Bisectors

**Goal**
- Use perpendicular bisectors to solve problems.

**VOCABULARY**

**Perpendicular bisector**
A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a perpendicular bisector.

**Equidistant**
A point is equidistant from two figures if the point is the same distance from each figure.

**Concurrent**
When three or more lines, rays, or segments intersect in the same point, they are called concurrent lines, rays, or segments.

**Point of concurrency**
The point of intersection of concurrent lines, rays, or segments is called the point of concurrency.

**Circumcenter**
The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle.

**THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM**

In a plane, if a point is on the perpendicular bisector of a segment, then it is **equidistant** from the endpoints of the segment.

If $\overline{CP}$ is the ⊥ bisector of $\overline{AB}$, then $CA = CB$.

**THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM**

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the **perpendicular bisector** of the segment.

If $DA = DB$, then $D$ lies on the ⊥ bisector of $\overline{AB}$. 
**Example 1** Use the Perpendicular Bisector Theorem

\( \overline{AC} \) is the perpendicular bisector of \( \overline{BD} \). Find \( AD \).

**Solution**

\[
AB = AD
\]

\[
4x = 7x - 6
\]

Substitute.

\[
x = 2
\]

Solve for \( x \).

\[
AD = 4x = 4(2) = 8
\]

**Example 2** Use perpendicular bisectors

In the diagram, \( \overrightarrow{KN} \) is the perpendicular bisector of \( \overline{JL} \).

a. What segment lengths in the diagram are equal?

b. Is \( M \) on \( \overrightarrow{KN} \)?

**Solution**

a. \( \overrightarrow{KN} \) bisects \( \overline{JL} \), so \( NJ = NL \). Because \( K \) is on the perpendicular bisector of \( \overline{JL} \), \( KJ = KL \) by Theorem 5.2. The diagram shows that \( MJ = ML = 13 \).

b. Because \( MJ = ML \), \( M \) is equidistant from \( J \) and \( L \). So, by the Converse of the Perpendicular Bisector Theorem, \( M \) is on the perpendicular bisector of \( \overline{JL} \), which is \( \overrightarrow{KN} \).

**Checkpoint** In the diagram, \( \overrightarrow{JK} \) is the perpendicular bisector of \( \overline{GH} \).

1. What segment lengths are equal?

\( KG = KH, JG = JH, FG = FH \)

2. Find \( GH \).

\( GH = 4 \)
**THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE**

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $PD$, $PE$, and $PF$ are perpendicular bisectors, then $PA = PB = PC$.

Example 3  
**Use the concurrency of perpendicular bisectors**

**Football** Three friends are playing catch. You want to join and position yourself so that you are the same distance from your friends. Find a location for you to stand.

**Solution**

Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points $A$, $B$, and $C$ and connect those points to draw $\triangle ABC$. Then use a ruler and a protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency $D$ is a location for you to stand.

**Checkpoint** Complete the following exercise.

3. In Example 3, your friend at location $A$ wants to move to a location that is the same distance from everyone else. Find a new location for $A$. 
5.3 Use Angle Bisectors of Triangles

Goal  • Use angle bisectors to find distance relationships.

VOCABULARY

Incenter  The point of concurrency of the three angle bisectors of a triangle is called the incenter of the triangle.

THEOREM 5.5: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $AD$ bisects $\angle BAC$ and $DB \perp AB$ and $DC \perp AC$, then $DB = DC$.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $DB \perp AB$ and $DC \perp AC$ and $DB = DC$, then $AD$ bisects $\angle BAC$.

Example 1  Use the Angle Bisector Theorems

Find the measure of $\angle CBE$.

Solution

Because $EC \perp BC$ and $ED \perp BD$ and $EC = ED = 21$, $BE$ bisects $\angle CBD$ by the Converse of the Angle Bisector Theorem. So, $m\angle CBE = m\angle DBE = 31^\circ$. 
Example 2  **Solve a real-world problem**

Web  A spider’s position on its web relative to an approaching fly and the opposite sides of the web forms congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?

**Solution**

The congruent angles tell you that the spider is on the **bisector** of $\angle LFR$. By the **Angle Bisector Theorem**, the spider is equidistant from $FL$ and $FR$.

So, the spider must move the **same distance** to reach each edge.

Example 3  **Use algebra to solve a problem**

For what value of $x$ does $P$ lie on the bisector of $\angle J$?

**Solution**

From the Converse of the Angle Bisector Theorem, you know that $P$ lies on the bisector of $\angle J$ if $P$ is equidistant from the sides of $\angle J$, so when $PK = PL$.

$$\frac{PK}{x+1} = \frac{PL}{2x-5}$$

Set segment lengths equal.

$$x + 1 = 2x - 5$$

Substitute expressions for segment lengths.

$$6 = x$$

Solve for $x$.

Point $P$ lies on the bisector of $\angle J$ when $x = 6$.

**THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE**

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If $AP$, $BP$, and $CP$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$. 
Example 4  Use the concurrency of angle bisectors

In the diagram, \( L \) is the incenter of \( \triangle FHJ \). Find \( LK \).

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter \( L \) is equidistant from the sides of \( \triangle FHJ \). So, to find \( LK \), you can find \( LI \) in \( \triangle LHI \). Use the Pythagorean Theorem.

\[
\begin{align*}
15^2 &= LI^2 + 12^2 \\
81 &= LI^2 \\
9 &= LI
\end{align*}
\]

Because \( LI = LK \), \( LK = 9 \).

Checkpoint  In Exercises 1 and 2, find the value of \( x \).

1. \( (3x - 5)^\circ \)

\[
x = 10
\]

2. \( 7x + 3 \)

\[
x = 3
\]

3. Do you have enough information to conclude that \( AC \) bisects \( \angle DAB \)? Explain.

No, you must know that \( m\angle ABC = m\angle ADC = 90^\circ \) before you can conclude that \( AC \) bisects \( \angle DAB \).

4. In Example 4, suppose you are not given \( HL \) or \( HI \), but you are given that \( JL = 25 \) and \(JI = 20 \). Find \( LK \).

\( LK = 15 \)
### 5.4 Use Medians and Altitudes

**Goal**  
• Use medians and altitudes of triangles.

<table>
<thead>
<tr>
<th><strong>VOCABULARY</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Median of a triangle</strong></td>
</tr>
</tbody>
</table>

**Centroid**  
The point of concurrency of the three medians of a triangle is the centroid.

**Altitude of a triangle**  
An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

**Orthocenter**  
The point at which the lines containing the three altitudes of a triangle intersect is called the orthocenter of the triangle.

**Theorem 5.8: Concurrency of Medians of a Triangle**

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

The medians of \( \triangle ABC \) meet at \( P \) and \( AP = \frac{2}{3} \overline{AE} \),  
\( BP = \frac{2}{3} \overline{BF} \), and \( CP = \frac{2}{3} \overline{CD} \).
Example 1 Use the centroid of a triangle

In \( \triangle FGH \), \( M \) is the centroid and \( GM = 6 \).
Find \( ML \) and \( GL \).

\[
\frac{GM}{3} = \frac{2}{3} GL \quad \text{Concurrency of Medians of a Triangle Theorem}
\]

\[
6 = \frac{2}{3} GL \quad \text{Substitute 6 for } GM.
\]

\[
9 = GL \quad \text{Multiply each side by the reciprocal, } \frac{3}{2}.
\]

Then \( ML = GL - \frac{GM}{3} = 9 - 6 = 3 \).
So, \( ML = 3 \) and \( GL = 9 \).

Checkpoint Complete the following exercise.

1. In Example 1, suppose \( FM = 10 \). Find \( MK \) and \( FK \).

\[
MK = 5, \ FK = 15
\]

Example 2 Find the centroid of a triangle

The vertices of \( \triangle JKL \) are \( J(1, 2) \), \( K(4, 6) \), and \( L(7, 4) \).
Find the coordinates of the centroid \( P \) of \( \triangle JKL \).

Sketch \( \triangle JKL \). Then use the Midpoint Formula to find the midpoint \( M \) of \( JL \) and sketch median \( KM \).

\[
M \left( \frac{1 + 7}{2}, \frac{2 + 4}{2} \right) = M(4, 3)
\]

The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex \( K \) to point \( M \) is \( 6 - 3 = 3 \) units. So, the centroid is \( \frac{2}{3} (3) = 2 \) units down from \( K \) on \( KM \).

The coordinates of the centroid \( P \) are \( (4, 6 - 2) \), or \( (4, 4) \).
THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

The lines containing the altitudes of a triangle are concurrent. The lines containing \(AF, BE,\) and \(CD\) meet at \(G.\)

Example 3

Find the orthocenter

Find the orthocenter \(P\) in the triangle.

a. 

b. 

Solution

a. 

b. 

Checkpoint Complete the following exercises.

2. In Example 2, where do you need to move point \(K\) so that the centroid is \(P(4, 5)\)?

Point \(K\) should be moved to \((4, 9)\).

3. Find the orthocenter \(P\) in the triangle.
Prove that the altitude to the base of an isosceles triangle is a median.

Solution

Given: \( \triangle ABC \) is isosceles, with base \( \overline{AC} \).
\( BD \) is the altitude to base \( \overline{AC} \).

Prove: \( BD \) is a median of \( \triangle ABC \).

Proof: Legs \( \overline{AB} \) and \( \overline{CD} \) of \( \triangle ABC \) are congruent. \( \angle ADB \) and \( \angle CDB \) are congruent right angles because \( BD \) is the altitude to \( \overline{AC} \). Also, \( BD \equiv BD \). Therefore, \( \triangle ADB \equiv \triangle CDB \) by the HL Congruence Theorem. \( \overline{AD} \equiv \overline{CD} \) because corresponding parts of congruent triangles are congruent. So, \( D \) is the midpoint of \( \overline{AC} \) by definition. Therefore, \( BD \) intersects \( \overline{AC} \) at its midpoint, and \( BD \) is a median of \( \triangle ABC \).

Checkpoint: Complete the following exercise.

4. Prove that the altitude \( BD \) in Example 4 is also an angle bisector.

Proof: Legs \( \overline{AB} \) and \( \overline{CB} \) of \( \triangle ABC \) are congruent. \( \angle ADB \) and \( \angle CDB \) are congruent right angles because \( BD \) is the altitude to \( \overline{AC} \). Also, \( BD \equiv BD \). Therefore, \( \triangle ADB \equiv \triangle CDB \) by the HL Congruence Theorem. \( \angle ABD \equiv \angle CBD \) because corresponding parts of congruent triangles are congruent. Therefore, \( BD \) bisects \( \angle ABC \), and \( BD \) is an angle bisector.
5.5 Use Inequalities in a Triangle

**Goal**
- Find possible side lengths of a triangle.

**Your Notes**

Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?

**Solution**

The longest side and largest angle are opposite each other.

The shortest side and smallest angle are opposite each other.

**THEOREM 5.10**

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

AB > BC, so \( \angle C > \angle A \).

**THEOREM 5.11**

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

\( \angle A > \angle C \), so \( BC > AB \).
Boating A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about 55° and one is about 24°. What is the angle measure of the path made at the cave?

Solution

The cave is opposite the longest side so, by Theorem 5.10, the cave angle is the largest angle.

The angle measures sum to 180°, so the third angle measure is \(180° - (55° + 24°) = 101°\).

The angle measure made at the cave is \(101°\).

Checkpoint Complete the following exercises.

1. List the sides of \(\triangle PQR\) in order from shortest to longest.
   \(QR, PQ, PR\)

2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.
THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

\[ AB + BC > AC \]
\[ AC + BC > AB \]
\[ AB + AC > BC \]

Example 3  Find possible side lengths

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

Solution

Let \( x \) represent the length of the third side. Draw diagrams to help visualize the small and large values of \( x \). Then use the Triangle Inequality Theorem to write and solve inequalities.

Small values of \( x \)

\[ x + 10 > 14 \]
\[ x > 4 \]

Large values of \( x \)

\[ 10 + 14 > x \]
\[ 24 > x \]

The length of the third side must be greater than 4 and less than 24.

Checkpoint  Complete the following exercise.

3. A triangle has one side of 23 meters and another of 17 meters. Describe the possible lengths of the third side.

The length of the third side must be greater than 6 meters and less than 40 meters.
**THEOREM 5.13: HINGE THEOREM**

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

**THEOREM 5.14: CONVERSE OF THE HINGE THEOREM**

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.
Given that $AD \cong BC$, how does $\angle 1$ compare to $\angle 2$?

**Solution**

You are given that $AD \cong BC$ and you know that $BD \cong BD$ by the Reflexive Property. Because $34 > 33$, $AB > CD$. So, two sides of $\triangle ADB$ are congruent to two sides of $\triangle CBD$ and the third side in $\triangle ADB$ is **longer**.

By the Converse of the Hinge Theorem, $m\angle 1 > m\angle 2$.

**Example 2**  
**Solve a multi-step problem**

**Travel** Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns $80^\circ$ toward east for 3 mi. Which car is farther from the mall?

Draw a diagram. The distance driven and the distance back to the mall form two triangles, with **congruent** 5 mile sides and **congruent** 3 mile sides. Add the third side to the diagram.

Use linear pairs to find the included angles of $90^\circ$ and $100^\circ$.

Because $100^\circ > 90^\circ$, Car **B** is farther from the mall than Car A by the **Hinge Theorem**.

**HOW TO WRITE AN INDIRECT (Proof by Contradiction)**

**Step 1** Identify the statement you want to prove. Assume temporarily that this statement is **false** by assuming that the opposite is **true**.

**Step 2** Reason logically until you reach a contradiction.

**Step 3** Point out that the desired conclusion must be **true** because the contradiction proves the temporary assumption **false**.
Example 3  Write an indirect proof

Write an indirect proof to show that an odd number is not divisible by 6.

Given  \( x \) is an odd number.
Prove  \( x \) is not divisible by 6.

Solution

Step 1  Assume temporarily that \( x \) is divisible by 6.
This means that \( \frac{x}{6} = n \) for some whole number \( n \).
So, multiplying both sides by 6 gives \( x = 6n \).

Step 2  If \( x \) is odd, then, by definition, \( x \) cannot be divided evenly by \( 2 \). However, \( x = 6n \) so \( \frac{x}{2} = \frac{6n}{2} = 3n \). We know that \( 3n \) is a whole number because \( n \) is a whole number, so \( x \) can be divided evenly by \( 2 \). This contradicts the given statement that \( x \) is odd.

Step 3  Therefore, the assumption that \( x \) is divisible by 6 is false, which proves that \( x \) is not divisible by 6.

Checkpoint  Complete the following exercises.

1. If \( m\angle ADB > m\angle CDB \) which is longer, \( AB \) or \( CB \)?
   \( AB \) is longer.

2. In Example 2, car C leaves the mall and goes 5 miles due west, then turns 85° toward south for 3 miles. Write the cars in order from the car closest to the mall to the car farthest from the mall.
   car A, car C, car B

3. Suppose you wanted to prove the statement “If \( x + y \neq 5 \) and \( y = 2 \), then \( x \neq 3 \).” What temporary assumption could you make to prove the conclusion indirectly?
   You can temporarily assume that \( x = 3 \).
Words to Review

Give an example of the vocabulary word.

<table>
<thead>
<tr>
<th>Midsegment of a triangle</th>
<th>Coordinate proof</th>
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</thead>
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<tr>
<td><img src="image1" alt="Midsegment of a triangle" /></td>
<td>A type of proof that involves placing geometric figures in a coordinate plane.</td>
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</table>

<table>
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<th>Perpendicular bisector</th>
<th>Equidistant</th>
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<td>Point $C$ is equidistant from point $A$ and point $B$.</td>
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<td><img src="image5" alt="Circumcenter" /></td>
<td><img src="image6" alt="Incenter" /></td>
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</tbody>
</table>
### Indirect proof

To use an indirect proof, assume that the original statement is false and that the opposite is true. If this assumption leads to a contradiction, then the assumption must be false and the original statement must be true.

---

**Review your notes and Chapter 5 by using the Chapter Review on pages 359–362 of your textbook.**
Ratios, Proportions, and the Geometric Mean

Goal • Solve problems by writing and solving proportions.

VOCABULARY

Ratio of \(a\) to \(b\)  If \(a\) and \(b\) are two numbers or quantities and \(b \neq 0\), then the ratio of \(a\) to \(b\) is \(\frac{a}{b}\).

Proportion An equation that states that two ratios are equal is a proportion.

Means, extremes In the proportion \(\frac{a}{b} = \frac{c}{d}\), \(b\) and \(c\) are the means, and \(a\) and \(d\) are the extremes.

Geometric mean The geometric mean of two positive numbers \(a\) and \(b\) is the positive number \(x\) that satisfies \(\frac{a}{x} = \frac{x}{b}\).

Example 1  Simplify ratios

Simplify the ratio. (See Table of Measures, p. 921)

a. 76 cm : 8 cm

Solution

a. Write 76 cm : 8 cm as \(\frac{76\text{ cm}}{8\text{ cm}}\). Then divide out the units and simplify.

\[
\frac{76\text{ cm}}{8\text{ cm}} = \frac{19}{2} = 19 : 2
\]

b. To simplify a ratio with unlike units, multiply by a conversion factor.

\[
\frac{4\text{ ft}}{24\text{ in.}} = \frac{4\text{ ft}}{24\text{ in.}} \cdot \frac{12\text{ in.}}{1\text{ ft}} = \frac{48}{24} = \frac{2}{1}
\]
Example 2  Use a ratio to find a dimension

Painting  You are painting barn doors. You know that the perimeter of the doors is 64 feet and that the ratio of the length to the height is 3 : 5. Find the area of the doors.

Solution

Step 1  Write expressions for the length and height. Because the ratio of the length to height is 3 : 5, you can represent the length by \( \frac{3}{5} x \) and the height by \( \frac{5}{3} x \).

Step 2  Solve an equation to find \( x \).

\[
2l + 2w = P
\]
\[
2\left(\frac{3}{5} x\right) + 2\left(\frac{5}{3} x\right) = 64
\]
\[
\frac{16}{x} = 64
\]
\[
x = \frac{4}{16}
\]

Step 3  Evaluate the expressions for the length and height. Substitute the value of \( x \) into each expression.

Length: \( \frac{3}{5} x = \frac{3}{5} \times \frac{4}{16} = \frac{12}{20} \)

Height: \( \frac{5}{3} x = \frac{5}{3} \times \frac{4}{16} = \frac{20}{20} \)

The doors are 12 feet long and 20 feet high, so the area is \( 12 \times 20 = 240 \text{ ft}^2 \).

Checkpoint  In Exercises 1 and 2, simplify the ratio.

1. 4 meters to 18 meters
   2 to 9

2. 33 yd : 9 ft
   11 : 1

3. The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5 : 2. Find the length and width of the table.
   length: 7.5 feet, width: 3 feet
**Example 3** Use extended ratios

The measures of the angles in \( \triangle BCD \) are in the extended ratio of 2:3:4. Find the measures of the angles.

**Solution**

Begin by sketching the triangle. Then use the extended ratio of 2:3:4 to label the measures as \( 2x^\circ \), \( 3x^\circ \), and \( 4x^\circ \).

\[
2x^\circ + 3x^\circ + 4x^\circ = 180^\circ \quad \text{Triangle Sum Theorem}
\]

\[
9x = 180 \quad \text{Combine like terms.}
\]

\[
x = \frac{20}{9}
\]

Divide each side by 9.

The angle measures are \( 2\left(\frac{20}{9}\right) = \frac{40}{9} \), \( 3\left(\frac{20}{9}\right) = \frac{60}{9} \), and \( 4\left(\frac{20}{9}\right) = \frac{80}{9} \).

**Checkpoint** Complete the following exercise.

4. A triangle’s angle measures are in the extended ratio of 1:4:5. Find the measures of the angles.

\( 18^\circ, 72^\circ, 90^\circ \)

**A PROPERTY OF PROPORTIONS**

1. Cross Products Property In a proportion, the product of the extremes equals the product of the means.

\[
\frac{a}{b} = \frac{c}{d} \quad \text{where} \quad b \neq 0 \quad \text{and} \quad d \neq 0, \quad \text{then} \quad ad = bc.
\]

\[
\frac{2}{3} = \frac{4}{6} \quad \text{so} \quad 3 \cdot \frac{4}{6} = \frac{12}{2} \cdot \frac{6}{2} = 12
\]
Example 4  Solve proportions

Solve the proportion.

a. \( \frac{3}{4} = \frac{x}{16} \)

Original proportion

Cross Products Property

Multiply.

Divide each side by \( 4 \).

b. \( \frac{3}{x + 1} = \frac{2}{x} \)

Original proportion

Cross Products Property

Distributive Property

Subtract \( 2x \) from each side.

Example 5  Solve a real-world problem

Bowling  You want to find the total number of rows of boards that make up 24 lanes at a bowling alley. You know that there are 117 rows in 3 lanes. Find the total number of rows of boards that make up the 24 lanes.

Solution

Write and solve a proportion involving two ratios that compare the number of rows with the number of lanes.

Write proportion.

Cross Products Property

Simplify.

There are \( 936 \) rows of boards that make up the 24 lanes.

GEOMETRIC MEAN

The geometric mean of two positive numbers \( a \) and \( b \) is the positive number \( x \) that satisfies \( \frac{a}{x} = \frac{x}{b} \).

So, \( x^2 = ab \) and \( x = \sqrt{ab} \).
Example 6  
**Find a geometric mean**

Find the geometric mean of 16 and 48.

**Solution**

\[ x = \sqrt{ab} \]  
Definition of geometric mean

\[ = \sqrt{16 \cdot 48} \]  
Substitute 16 for \( a \) and 48 for \( b \).

\[ = \sqrt{16 \cdot 16 \cdot 3} \]  
Factor.

\[ = 16\sqrt{3} \]  
Simplify.

The geometric mean of 16 and 48 is \( 16\sqrt{3} \approx 27.7 \).

**Checkpoint** Complete the following exercises.

5. Solve \( \frac{8}{y} = \frac{2}{5} \).
   
   \[ y = 20 \]

6. Solve \( \frac{x - 3}{3} = \frac{2x}{9} \).
   
   \[ x = 9 \]

7. A small gymnasium contains 10 sets of bleachers. You count 192 spectators in 3 sets of bleachers and the spectators seem to be evenly distributed. Estimate the total number of spectators.
   
   about 640 spectators

8. Find the geometric mean of 14 and 16.
   
   \( 4\sqrt{14} \approx 15.0 \)
6.2 Use Proportions to Solve Geometry Problems

Goal • Use proportions to solve geometry problems.

VOCABULARY

Scale drawing  A scale drawing is a drawing that is the same shape as the object it represents.

Scale  The scale is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

ADDITIONAL PROPERTIES OF PROPORTIONS

2. Reciprocal Property  If two ratios are equal, then their reciprocals are also equal.

   If \( \frac{a}{b} = \frac{c}{d} \) then \( \frac{b}{a} = \frac{d}{c} \).

3. If you interchange the means of a proportion, then you form another true proportion.

   If \( \frac{a}{b} = \frac{c}{d} \) then \( \frac{a}{c} = \frac{b}{d} \).

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

   If \( \frac{a}{b} = \frac{c}{d} \) then \( \frac{a + b}{b} = \frac{c + d}{d} \).
Example 1  Use properties of proportions

In the diagram, \( \frac{AC}{DF} = \frac{BC}{EF} \). Write four true proportions.

Because \( \frac{AC}{DF} = \frac{BC}{EF} \), then \( \frac{12}{18} = \frac{9}{x} \).

Reciprocal Property: The reciprocals are equal, so \( \frac{12}{18} = \frac{x}{9} \).

Property 3: You can interchange the means, so \( \frac{12}{9} = \frac{18}{x} \).

Property 4: You can add the denominators to the numerators, so \( \frac{30}{18} = \frac{9 + x}{x} \).

Example 2  Use proportions with geometric figures

In the diagram, \( \frac{JL}{LH} = \frac{JK}{KG} \). Find \( JH \) and \( JL \).

\[
\frac{JL}{LH} = \frac{JK}{KG}
\]

\[
\frac{JL + LH}{LH} = \frac{JK + KG}{KG}
\]

\[
\frac{x}{2} = \frac{15 + 5}{5}
\]

\[
5x = 2(15 + 5)
\]

\[
x = 8
\]

So \( JH = 8 \) and \( JL = 8 - 2 = 6 \).

Checkpoint  Complete the following exercises.

1. In Example 1, find the value of \( x \).
   \[ x = 13.5 \]

2. In Example 2,
   \[
   \frac{KL}{GH} = \frac{JK}{JG}
   \]
   Find \( GH \).
   \[ GH = 16 \]
Example 3  Find the scale of a drawing

Keys  The length of the key in the scale drawing is 7 centimeters. The length of the actual key is 4 centimeters. What is the scale of the drawing?

Solution
To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

\[
\frac{\text{length in drawing}}{\text{length of key}} = \frac{7 \text{ cm}}{4 \text{ cm}} = \frac{7 \div 4}{4 \div 4} = \frac{1.75}{1}
\]

The scale of the drawing is \(1.75 \text{ cm} : 1 \text{ cm}\).

Checkpoint  Complete the following exercise.

3. In Example 3, suppose the length of the key in the scale drawing is 6 centimeters. Find the new scale of the drawing.

\(1.5 \text{ cm} : 1 \text{ cm}\)

Example 4  Use a scale drawing

Maps  The scale of the map at the right is 1 inch : 8 miles. Find the actual distance from Westbrook to Cooley.

Solution
Use a ruler. The distance from Westbrook to Cooley on the map is about 1.25 inches. Let \(x\) be the actual distance in miles.

\[
\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{8 \text{ mi}} \quad \text{Cross Products Property}
\]

\[
x = \frac{1.25(8)}{1} = \frac{10}{1} = 10 \quad \text{Simplify.}
\]

The actual distance from Westbrook to Cooley is about 10 miles.
Two landmarks are 130 miles from each other. The landmarks are 6.5 inches apart on a map. Find the scale of the map.

1 inch : 20 miles

Your friend has a model of the Sunsphere that is 5 inches tall. What is the approximate diameter of the dome on your friend’s model?

about 1.4 inches
6.3 Use Similar Polygons

Goal • Use proportions to identify similar polygons.

Your Notes

VOCABULARY

Similar polygons Two polygons are similar polygons if corresponding angles are congruent and corresponding side lengths are proportional.

Scale factor of two similar polygons If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the scale factor.

Example 1 Use similarity statements

In the diagram, \( \triangle ABC \sim \triangle DEF \).

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a statement of proportionality.

Solution

a. \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), \( \angle C \cong \angle F \)

b. \( \frac{AB}{DE} = \frac{10}{15} = \frac{2}{3} \), \( \frac{BC}{EF} = \frac{8}{12} = \frac{2}{3} \), \( \frac{CA}{FD} = \frac{12}{18} = \frac{2}{3} \)

c. The ratios in part (b) are equal, so \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \).

Checkpoint Complete the following exercise.

1. Given \( \triangle PQR \sim \triangle XYZ \), list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

\( \angle P \cong \angle X \), \( \angle Q \cong \angle Y \), \( \angle R \cong \angle Z \); \( \frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX} \)
Your Notes

Example 2  Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of \(ABCD\) to \(JKLM\).

Solution

Step 1 Identify pairs of congruent angles.
From the diagram, you can see that \(\angle B \cong \angle K\), \(\angle C \cong \angle L\), and \(\angle D \cong \angle M\). Angles \(A\) and \(J\) are right angles, so \(\angle A \cong \angle J\). So, the corresponding angles are congruent.

Step 2 Show that corresponding side lengths are proportional.
\[
\begin{align*}
\frac{AB}{JK} &= \frac{8}{14} = \frac{4}{7} \\
\frac{BC}{KL} &= \frac{8}{14} = \frac{4}{7} \\
\frac{CD}{LM} &= \frac{8}{14} = \frac{4}{7} \\
\frac{AD}{JM} &= \frac{12}{21} = \frac{4}{7}
\end{align*}
\]

The ratios are equal, so the corresponding side lengths are proportional.

So \(ABCD \sim JKLM\). The scale factor of \(ABCD\) to \(JKLM\) is \(\frac{4}{7}\).

Example 3  Use similar polygons

In the diagram, \(\triangle BCD \sim \triangle RST\). Find the value of \(x\).

Solution

The triangles are similar, so the corresponding side lengths are proportional.

\[
\frac{BC}{RS} = \frac{CD}{ST}
\]

Write proportion.

\[
\frac{12}{24} = \frac{13}{x}
\]

Substitute.

\[
12x = 312
\]

Cross Products Property

\[
x = 26
\]

Solve for \(x\).
2. What is the scale factor of $LMNP$ to $FGHJ$?

\[
\frac{4}{5}
\]

3. Find the value of $x$.

32

**THEOREM 6.1: PERIMETERS OF SIMILAR POLYGONS**

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then

\[
\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}.
\]

**Example 4**  
**Find perimeters of similar figures**

Basketball A larger cement court is being poured for a basketball hoop in place of a smaller one. The court will be 20 feet wide and 25 feet long. The old court was similar in shape, but only 16 feet wide.

a. Find the scale factor of the new court to the old court.

b. Find the perimeters of the new court and the old court.

**Solution**

a. Because the new court will be similar to the old court, the scale factor is the ratio of the widths, \( \frac{20}{16} = \frac{5}{4} \).

b. The new court’s perimeter is \( 2(20) + 2(25) = 90 \) feet. Use Theorem 6.1 to find the perimeter $x$ of the old court.

\[
\frac{90}{x} = \frac{5}{4} \quad \text{Use Theorem 6.1 to write a proportion.}
\]

\[
x = \frac{72}{4} = 72 \quad \text{Simplify.}
\]

The perimeter of the old court was 72 feet.
CORRESPONDING LENGTHS IN SIMILAR POLYGONS

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the _scale factor_ of the similar polygons.

**Example 5 Use a scale factor**

In the diagram, \( \triangle FGH \sim \triangle JGK \).

Find the length of the altitude \( GL \).

**Solution**

First, find the scale factor of \( \triangle FGH \) to \( \triangle JGK \).

\[
\frac{FH}{JK} = \frac{8 + 8}{5 + 5} = \frac{16}{10} = \frac{8}{5}
\]

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

\[
\frac{GL}{GM} = \frac{8}{5} \quad \text{Write proportion.}
\]

\[
\frac{GL}{14} = \frac{8}{5} \quad \text{Substitute 14 for } GM.
\]

\[
GL = 22.4 \quad \text{Multiply each side by 14 and simplify.}
\]

The length of altitude \( GL \) is 22.4.

**Checkpoint** In the diagrams, \( \triangle PQR \sim \triangle WXY \).

4. Find the perimeter of \( \triangle WXY \).
   The perimeter of \( \triangle WXY \) is 120.

5. Find the length of median \( QS \).
   \( QS = 26 \)
Find the eighth and ninth terms of the Fibonacci sequence.

Solution
Each term of the Fibonacci sequence after the second term is the sum of the previous two terms.

<table>
<thead>
<tr>
<th>Term</th>
<th>Calculation</th>
<th>Terms Used</th>
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<tbody>
<tr>
<td>3rd</td>
<td>1 + 1 = 2</td>
<td>1st and 2nd terms</td>
</tr>
<tr>
<td>4th</td>
<td>1 + 2 = 3</td>
<td>2nd and 3rd terms</td>
</tr>
<tr>
<td>5th</td>
<td>2 + 3 = 5</td>
<td>3rd and 4th terms</td>
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<tr>
<td>6th</td>
<td>3 + 5 = 8</td>
<td>4th and 5th terms</td>
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<td>7th</td>
<td>5 + 8 = 13</td>
<td>5th and 6th terms</td>
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<td>8th</td>
<td>8 + 13 = 21</td>
<td>6th and 7th terms</td>
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<tr>
<td>9th</td>
<td>13 + 21 = 34</td>
<td>7th and 8th terms</td>
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</table>

Example 1 Find terms in the Fibonacci sequence
Find the eighth and ninth terms of the Fibonacci sequence.

Solution
Each term of the Fibonacci sequence after the second term is the sum of the previous two terms.

Checkpoint Find terms of the Fibonacci sequence.

1. What are the tenth and eleventh terms of the Fibonacci sequence?
   
   55; 89
Example 2 Find ratios of terms in the Fibonacci sequence

Find the ratios of consecutive terms in the Fibonacci sequence using the seventh, eighth, and ninth terms. Round to the nearest 0.001.

Solution

The seventh, eighth, and ninth terms of the Fibonacci sequence are 13, 21, and 34.

\[
\frac{8\text{th term}}{7\text{th term}} = \frac{21}{13} = 1.615 \quad \frac{9\text{th term}}{8\text{th term}} = \frac{34}{21} = 1.619
\]

Checkpoint Complete the following exercise.

2. Which two consecutive terms of the Fibonacci sequence will give you a ratio of approximately 1.618?
   ninth and tenth terms

Example 3 The golden rectangle

Show that the figure is nearly a golden rectangle.

Solution

For a rectangle to be a golden rectangle,

\[
\frac{\text{length}}{\text{width}} = \frac{\text{width} + \text{length}}{\text{length}} \approx 1.618.
\]

For the figure shown,

\[
\frac{6.5}{4} = 1.625, \text{ and } \frac{6.5 + 4}{6.5} \approx 1.615.
\]

Yes, the figure is nearly a golden rectangle.

Checkpoint Complete the following exercise.

3. Is a 15 m \times 13 m rectangle nearly a golden rectangle?
   No
**Goal**  • Use the AA Similarity Postulate.

**POSTULATE 22: ANGLE-ANGLE (AA) SIMILARITY POSTULATE**

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

**Example 1**  **Use the AA Similarity Postulate**

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

**Solution**

Because they are both right angles, \( \angle B \) and \( \angle E \) are congruent.

By the Triangle Sum Theorem,

\[
\angle B + 90^\circ + m\angle A = 180^\circ, \text{ so } m\angle A = 22^\circ.
\]

Therefore, \( \angle A \) and \( \angle D \) are congruent.

So, \( \triangle ABC \sim \triangle DEF \) by the **AA Similarity Postulate**.

**Checkpoint** Determine whether the triangles are similar. If they are, write a similarity statement.

1. \( \triangle FGH \sim \triangle RQP \)

2. not similar
Example 2  
**Show that triangles are similar**

Show that the two triangles are similar.

a. $\triangle RTV$ and $\triangle RQS$

b. $\triangle LMN$ and $\triangle NOP$

**Solution**

a. You may find it helpful to redraw the triangles separately.

Because $\angle RTV$ and $\angle Q$ both equal 49°, $\angle RTV \cong \angle Q$. By the Reflexive Property, $\angle R \cong \angle R$.

So, $\triangle RTV \sim \triangle RQS$ by the **AA Similarity Postulate**.

b. The diagram shows $\angle L \cong \angle ONP$. It also shows that $MN \parallel OP$ so $\angle LNM \cong \angle P$ by the Corresponding Angles Postulate.

So, $\triangle LMN \sim \triangle NOP$ by the **AA Similarity Postulate**.

**Checkpoint**  Complete the following exercise.

3. Show that $\triangle BCD \sim \triangle EFD$.

Because they are both right angles, $\angle C \cong \angle F$.

You know that $\angle CDB \cong \angle FDE$ by the Vertical Angles Congruence Theorem.

So, $\triangle BCD \sim \triangle EFD$ by the AA Similarity Postulate.
Example 3  Using similar triangles

Height  A lifeguard is standing beside the lifeguard chair on a beach. The lifeguard is 6 feet 4 inches tall and casts a shadow that is 48 inches long. The chair casts a shadow that is 6 feet long. How tall is the chair?

Solution

The lifeguard and the chair form sides of two right triangles with the ground, as shown below. The sun’s rays hit the lifeguard and the chair at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.

\[
\frac{6 \text{ ft}}{48 \text{ in.}} = \frac{x \text{ ft}}{6 \text{ ft}}
\]

You can use a proportion to find the height \(x\). Write 6 feet 4 inches as \(\frac{76}{12}\) inches so you can form two ratios of feet to inches.

\[
\frac{x}{76 \text{ in.}} = \frac{6}{48 \text{ in.}}
\]

\[
48x = 456
\]

\[
x = 9.5
\]

The chair is 9.5 feet tall.

Checkpoint  Complete the following exercise.

4. In Example 3, how long is the shadow of a person that is 4 feet 9 inches tall?

3 feet
Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?

**Solution**

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

<table>
<thead>
<tr>
<th>Shortest sides</th>
<th>Longest sides</th>
<th>Remaining sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB$</td>
<td>$8$</td>
<td>$4$</td>
</tr>
<tr>
<td>$DE$</td>
<td>$4$</td>
<td>$2$</td>
</tr>
</tbody>
</table>

The ratios are **not all equal**, so $\triangle ABC$ and $\triangle DEF$ are **not similar**.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

<table>
<thead>
<tr>
<th>Shortest sides</th>
<th>Longest sides</th>
<th>Remaining sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB$</td>
<td>$8$</td>
<td>$16$</td>
</tr>
<tr>
<td>$GH$</td>
<td>$16$</td>
<td>$2$</td>
</tr>
</tbody>
</table>

All the ratios are **equal**, so $\triangle ABC \sim \triangle GHJ$.
**Example 2**

*Use the SSS Similarity Theorem*

Find the value of $x$ that makes $\triangle ABC \sim \triangle DEF$.

**Solution**

**Step 1** Find the value of $x$ that makes corresponding side lengths proportional.

$$\frac{4}{10} = \frac{x - 3}{20}$$

Write proportion.

$$4 \cdot 20 = 10(x - 3)$$

Cross Products Property

$$80 = 10x - 30$$

Simplify.

$$x = 11$$

Solve for $x$.

**Step 2** Check that the side lengths are proportional when $x = 11$.

$$AB = x - 3 = 8$$

$$DF = 2x + 3 = 25$$

$$\frac{BC}{EF} \overset{?}{=} \frac{AB}{DE} \quad \frac{4}{10} = \frac{8}{20} \checkmark$$

$$\frac{BC}{EF} \overset{?}{=} \frac{AC}{DF} \quad \frac{4}{10} = \frac{10}{25} \checkmark$$

When $x = 11$, the triangles are similar by the **SSS Similarity Theorem**.

**Checkpoint** Complete the following exercises.

1. Which of the three triangles are similar?

   $\triangle PQR \sim \triangle ZXY$

2. Suppose $AB$ is not given in $\triangle ABC$. What value of $AB$ would make $\triangle ABC$ similar to $\triangle QRP$?

   $$AB = 36$$
**THEOREM 6.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM**

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$, and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

**Example 3**  
**Use the SAS Similarity Theorem**

**Birdfeeder** You are drawing a design for a birdfeeder. Can you construct the top so it is similar to the bottom using the angle measure and lengths shown?

**Solution**

Both $m\angle B$ and $m\angle E$ equal $87^\circ$, so $\angle B \cong \angle E$. Next, compare the ratios of the lengths of the sides that include $\angle B$ and $\angle E$.

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{32}{20} = \frac{8}{5}
\]

The lengths of the sides that include $\angle B$ and $\angle E$ are proportional.

So, by the **SAS Similarity Theorem**, $\triangle ABC \sim \triangle DEF$.

Yes, you can make the top similar to the bottom.

**Checkpoint** Complete the following exercise.

3. In Example 3, suppose you use equilateral triangles on the top and bottom. Are the top and bottom similar? *Explain.*

   Yes, the top and bottom are similar. If the side length of the top is $a$ and the side length of the bottom is $b$, the ratios of the side lengths are $\frac{a}{b}$ and the angles are all $60^\circ$. The triangles are similar by SAS or SSS.
TRIANGLE SIMILARITY POSTULATE AND THEOREMS

AA Similarity Postulate  If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem  If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem  If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Example 4  Choose a method

Tell what method you would use to show that the triangles are similar.

Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{QR}{RS} = \frac{9}{12} = \frac{3}{4}$

Longer sides $\frac{PR}{RT} = \frac{18}{24} = \frac{3}{4}$

The corresponding side lengths are proportional. The included angles $\angle PRQ$ and $\angle TRS$ are congruent because they are vertical angles. So, $\triangle PQR \sim \triangle TSR$ by the SAS Similarity Theorem.

Checkpoint  Complete the following exercise.

Homework

4. Explain how to show $\triangle JKL \sim \triangle LKM$.

Show that the corresponding side lengths are proportional, then use the SSS Similarity Theorem to show $\triangle JKL \sim \triangle LKM$. 
In the diagram, $QS \parallel UT, RQ = 10, RS = 12, \text{ and } ST = 6$. What is the length of $QU$?

**Solution**

$$\frac{RQ}{QU} = \frac{RS}{ST} \quad \text{Triangle Proportionality Theorem}$$

$$\frac{10}{QU} = \frac{12}{6} \quad \text{Substitute.}$$

$$60 = 12 \cdot QU \quad \text{Cross Products Property}$$

$$5 = QU \quad \text{Divide each side by 12.}$$
**Example 2**  Solve a real-world problem

Aerodynamics  A spoiler for a remote controlled car is shown where \(AB = 31\) mm, \(BC = 19\) mm, \(CD = 27\) mm, and \(DE = 23\) mm. Explain why \(BD\) is not parallel to \(AE\).

**Solution**

Find and simplify the ratios of lengths determined by \(BD\).

\[
\frac{CD}{DE} = \frac{27}{23} \quad \frac{CB}{BA} = \frac{31}{19}
\]

Because \(\frac{27}{23} \neq \frac{31}{19}\), \(BD \not\parallel AE\).

**Checkpoint**  Complete the following exercises.

1. Find the length of \(KL\).

   \(KL = 33\)

2. Determine whether \(QT \parallel RS\).

   No; \(QT\) is not parallel to \(RS\).
**Example 3  Use Theorem 6.6**

**Farming** A farmer’s land is divided by a newly constructed interstate. The distances shown are in meters. Find the distance \(CA\) between the north border and the south border of the farmer’s land.

Use Theorem 6.6.

\[
\frac{CB}{BA} = \frac{DE}{EF}
\]

\[
\frac{CB + BA}{BA} = \frac{DE + EF}{EF}
\]

\[
\frac{CA}{2000} = \frac{3000 + 2500}{2500}
\]

\[
\frac{CA}{2000} = \frac{5500}{2500}
\]

\[
CA = \frac{4400}{2000}
\]

The distance between the north border and the south border is \(4400\) meters.
In the diagram, \( \angle DEG \cong \angle GEF \). Use the given side lengths to find the length of \( DG \).

**Solution**

Because \( EG \) is an angle bisector of \( \angle DEF \), you can apply Theorem 6.7. Let \( GD = x \). Then \( GF = \frac{14 - x}{2} \).

\[
\frac{GF}{GD} = \frac{EF}{ED}
\]

\[
\frac{14 - x}{x} = \frac{12}{8}
\]

Substitute.

\[
12x = 112 - 8x
\]

Cross Products Property

\[
x = 5.6
\]

Solve for \( x \).

**Checkpoint** Find the length of \( AB \).

3. \[
\begin{array}{c}
24 \\
27 \\
32 \\
\hline
24 + 27 + 32 = 83
\end{array}
\]

\( AB = 36 \)

4. \[
\begin{array}{c}
A \\
B \\
C \\
D \\
\hline
1 \\
\sqrt{2}
\end{array}
\]

\( AB = \sqrt{2} \)

**Homework**
6.7 Similarity Transformations and Coordinate Geometry

**Goal**
- Perform dilations.

**VOCABULARY**

Dilation A dilation is a transformation that stretches or shrinks a figure to create a similar figure.

Center of dilation In a dilation, a figure is enlarged or reduced with respect to a fixed point called the center of dilation.

Scale factor of a dilation The scale factor $k$ of a dilation is the ratio of a side length of the image to the corresponding side length of the original figure.

Reduction A dilation where $0 < k < 1$ is a reduction.

Enlargement A dilation where $k > 1$ is an enlargement.

**COORDINATE NOTATION FOR A DILATION**

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where $k$ is the scale factor.

If $0 < k < 1$, the dilation is a reduction. If $k > 1$, the dilation is an enlargement.
Example 1  Draw a dilation with a scale factor greater than 1

Draw a dilation of quadrilateral ABCD with vertices A(2, 0), B(6, -4), C(8, 2), and D(6, 4). Use a scale factor of \( \frac{1}{2} \).

First draw ABCD. Find the dilation of each vertex by multiplying its coordinates by \( \frac{1}{2} \). Then draw the dilation.

\[
(x, y) \rightarrow \left( \frac{1}{2}x, \frac{1}{2}y \right)
\]

\( A(2, 0) \rightarrow L(1, 0) \)

\( B(6, -4) \rightarrow M(3, -2) \)

\( C(8, 2) \rightarrow N(4, 1) \)

\( D(6, 4) \rightarrow P(3, 2) \)

Example 2  Verify that a figure is similar to its dilation

A triangle has the vertices A(2, -1), B(4, -1), and C(4, 2). The image of \( \triangle ABC \) after a dilation with a scale factor of 2 is \( \triangle DEF \).

a. Sketch \( \triangle ABC \) and \( \triangle DEF \).

b. Verify that \( \triangle ABC \) and \( \triangle DEF \) are similar.

Solution

a. The scale factor is greater than 1, so the dilation is an enlargement.

\[
(x, y) \rightarrow \left( \frac{1}{2}x, \frac{2}{2}y \right)
\]

\( A(2, -1) \rightarrow D(4, -2) \)

\( B(4, -1) \rightarrow E(8, -2) \)

\( C(4, 2) \rightarrow F(8, 4) \)

b. Because \( \angle B \) and \( \angle E \) are both right angles, \( \angle B = \angle E \). Show that the lengths of the sides that include \( \angle B \) and \( \angle E \) are proportional.

\[
\frac{AB}{DE} = \frac{BC}{EF} = \frac{2}{4} = \frac{3}{6}
\]

The lengths are proportional. So, \( \triangle ABC \sim \triangle DEF \) by the SAS Similarity Theorem.
Magnets You are making your own photo magnets. Your photo is 8 inches by 10 inches. The image on the magnet is 2.8 inches by 3.5 inches. What is the scale factor of the reduction?

Solution
The scale factor is the ratio of a side length of the magnet image to a side length of the original photo, or \( \frac{2.8 \text{ in.}}{8 \text{ in.}} \). In simplest form, the scale factor is \( \frac{7}{20} \).

Example 3 Find a scale factor

Sketch \( \triangle BCD \) and \( \triangle LMN \).

L(−4, −4), M(0, 4), N(4, 0)

Check your answer by sketching the triangle described in Example 3 with a scale factor of 4.

2. In Example 3, what is the scale factor of the reduction if your photo is 4 inches by 5 inches?

\( \frac{7}{10} \)
You want to create a quadrilateral $JKLM$ that is similar to quadrilateral $PQRS$. What are the coordinates of $M$?

**Solution**

Determine if $JKLM$ is a dilation of $PQRS$ by checking whether the same scale factor can be used to obtain $J$, $K$, and $L$ from $P$, $Q$, and $R$.

\[
(x, y) \rightarrow (kx, ky)
\]

\[
P(0, 3) \rightarrow J(0, 6) \quad k = 2
\]

\[
Q(2, 2) \rightarrow K(4, 4) \quad k = 2
\]

\[
R(2, 0) \rightarrow L(4, 0) \quad k = 2
\]

Because $k$ is the same in each case, the image is a **dilation** with a scale factor of **2**. So, you can use the scale factor to find the image $M$ of point $S$.

\[
S(5, 5) \rightarrow M(2 \cdot 5, 2 \cdot 5) = M(10, 10)
\]

**Checkpoint** Complete the following exercise.

3. You want to create a quadrilateral $QRST$ that is similar to quadrilateral $WXYZ$. What are the coordinates of $T$?

\[
T(10, 15)
\]
Words to Review

Give an example of the vocabulary word.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{4}{3} ) or ( 4 : 3 )</td>
<td>( \frac{x}{4} = \frac{3}{12} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Means</th>
<th>Extremes</th>
</tr>
</thead>
<tbody>
<tr>
<td>The means of ( \frac{a}{b} = \frac{c}{d} ) are ( b ) and ( c ).</td>
<td>The extremes of ( \frac{a}{b} = \frac{c}{d} ) are ( a ) and ( d ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometric mean</th>
<th>Scale drawings</th>
</tr>
</thead>
<tbody>
<tr>
<td>The geometric mean of two positive numbers ( a ) and ( b ) is the positive number ( x ) that satisfies ( \frac{a}{x} = \frac{x}{b} ).</td>
<td>A scale drawing is a drawing that is the same shape as the object it represents. A map is an example of a scale drawing.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
<th>Similar polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>The scale of a map is 1 inch to 25 miles.</td>
<td>( ABCD \sim FGHJ )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale factor of two similar polygons</th>
<th>Fibonacci sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>The scale factor of ( ABCD ) to ( FGHJ ) is ( \frac{1}{2} ).</td>
<td>( 1, 1, 2, 3, 5, 8, 13, \ldots )</td>
</tr>
</tbody>
</table>

Golden ratio 1.618...
<table>
<thead>
<tr>
<th>Golden rectangle</th>
<th>Dilation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Golden rectangle image" /></td>
<td><img src="image" alt="Dilation image" /></td>
</tr>
<tr>
<td>$1.618...$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Center of dilation</th>
<th>Scale factor of a dilation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Center of dilation image" /></td>
<td><img src="image" alt="Scale factor of a dilation image" /></td>
</tr>
<tr>
<td>The center of dilation is $(0, 0)$.</td>
<td>The scale factor of the dilation is $\frac{XY}{AB}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reduction</th>
<th>Enlargement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A dilation with a scale factor greater than 0 and less than 1 is a reduction.</td>
<td>A dilation with a scale factor greater than 1 is an enlargement.</td>
</tr>
</tbody>
</table>

Review your notes and Chapter 6 by using the Chapter Review on pages 435–438 of your textbook.